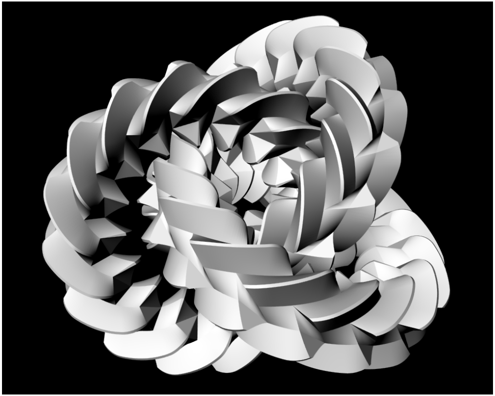
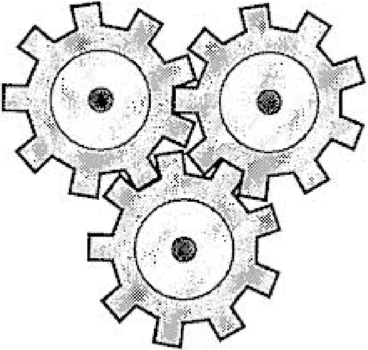


Henry Segerman
Oklahoma State University

Saul Schleimer
University of Warwick

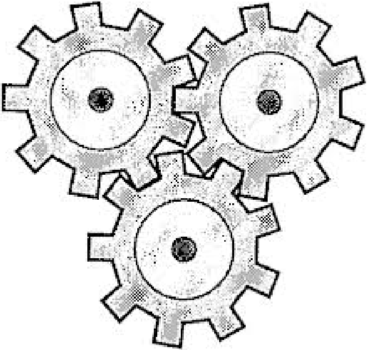
Triple gear

Manchester Metroshuttle advertisement, photo credit: Bill Beaty



Cooperative learning logo from the University of Saskatchewan.

Manchester Metroshuttle advertisement, photo credit: Bill Beaty



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Three pairwise meshing gears are usually frozen...

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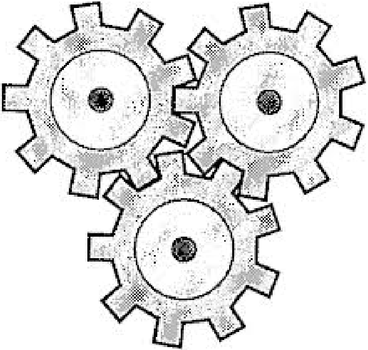
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Three pairwise meshing gears are usually frozen...

A challenge: Find a triple of pairwise meshing gears that moves!

"Umbilic Rolling Link" by Helaman Ferguson.

"Knotted Gear" by Oskar van Deventer.

Our solution is inspired by these "linked" gears.

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Our solution is inspired by these "linked" gears.

They have two "gears"; we want to do the same with three.

But we need to say what "the same" means...

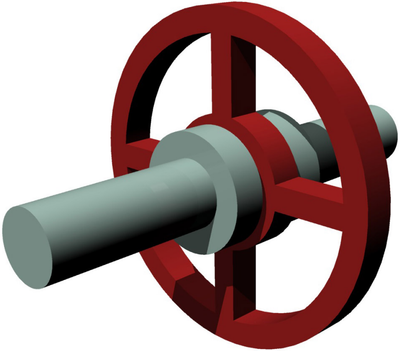
"Umbilic Rolling Link" by Helaman Ferguson.

"Knotted Gear" by Oskar van Deventer.

In both examples the gears are

Tracked: The gears can move relative to each other, but basically in only one way.

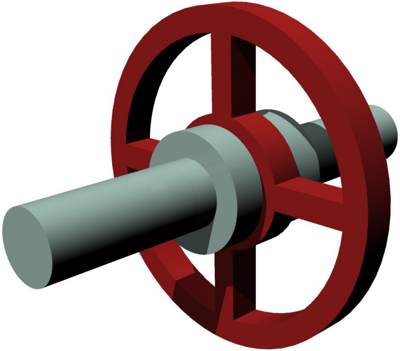
“Knotted Gear” by Oskar van Deventer.



A wheel on an axle.

Also they have no “gearbox”; everything is a gear.

“Knotted Gear” by Oskar van Deventer.



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For a wheel on an axle, the axle acts as a gearbox.

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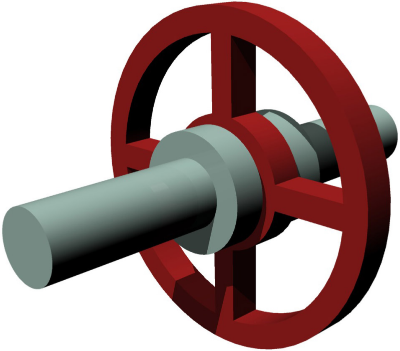

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For a wheel on an axle, the axle acts as a gearbox.

We rule this out via

Epicyclic: The movement of one gear in the frame of reference of another is not a rotation.

Axioms

So far we have

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- Symmetry: Any of the gears may be taken to any other by a rigid motion preserving the mechanism.

We want to construct a mechanism with three gears that satisfies these axioms.

If the gears could be separated, there would be too many ways for them to move - violating Tracked. So they have to be linked somehow.

They also have to be rings, that is round, so that when they rotate their shapes don't change too much.

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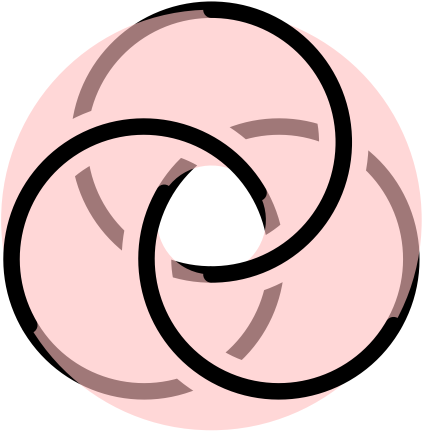

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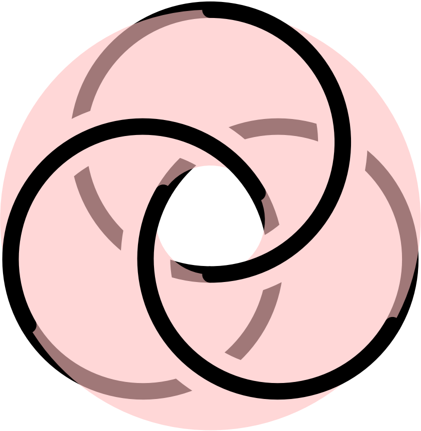
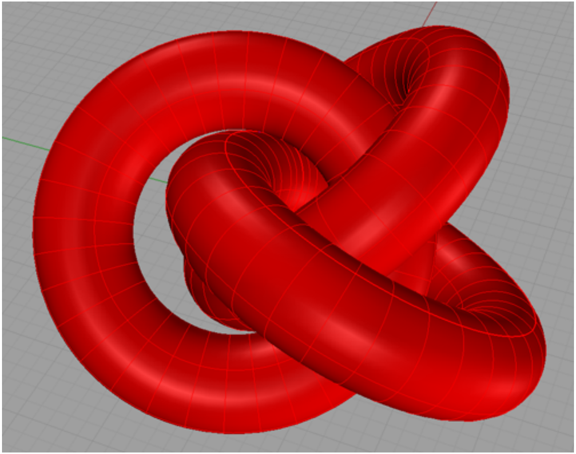


In fact there is only one symmetric way to do this: the three component Hopf link.

Try it! Take three round key-rings and link them all pairwise. Then you will have made the three-component Hopf link. Nothing else is possible!

To satisfy Tracked, the gears must remain in contact. To enforce this, we gradually inflate the three rings, letting them bump against each other while preserving the 3-fold symmetry, until they reach maximum thickness.



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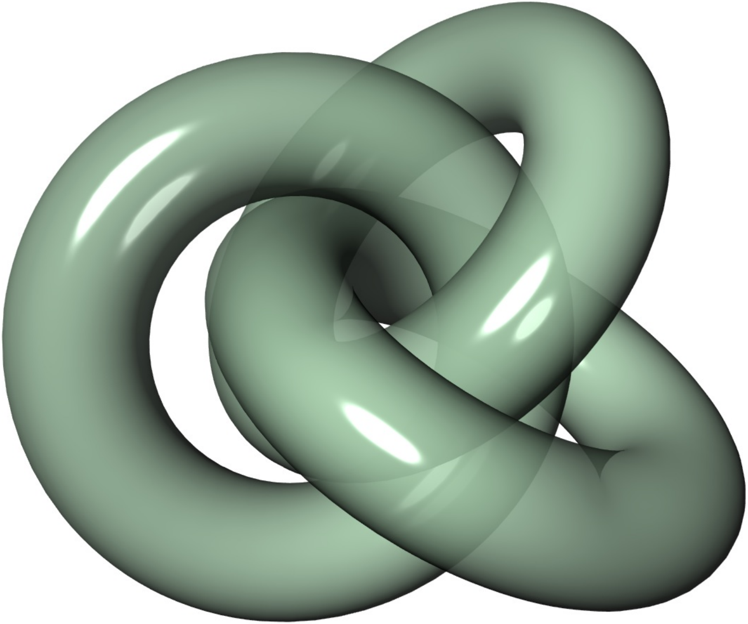
To stop them moving out of place, we design gear teeth.

To design the teeth, we investigate how the rings touch each other.



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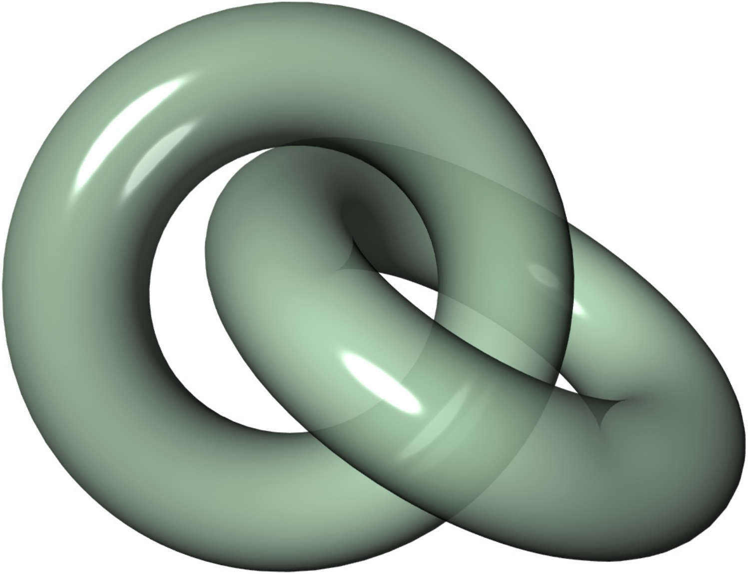

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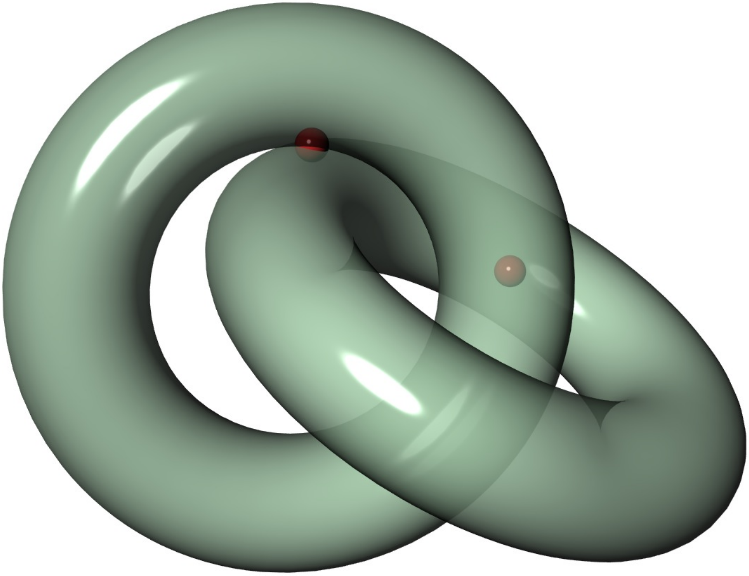

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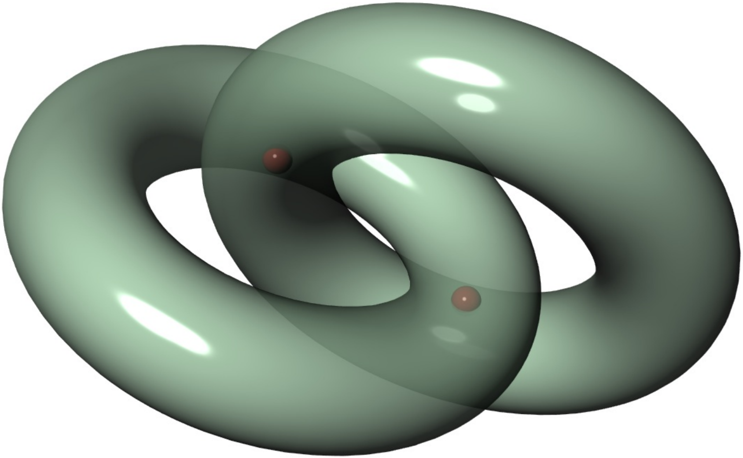

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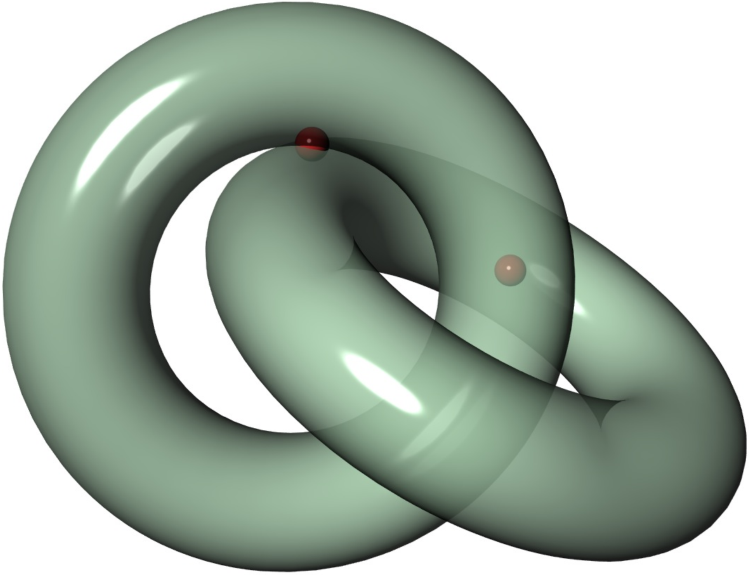

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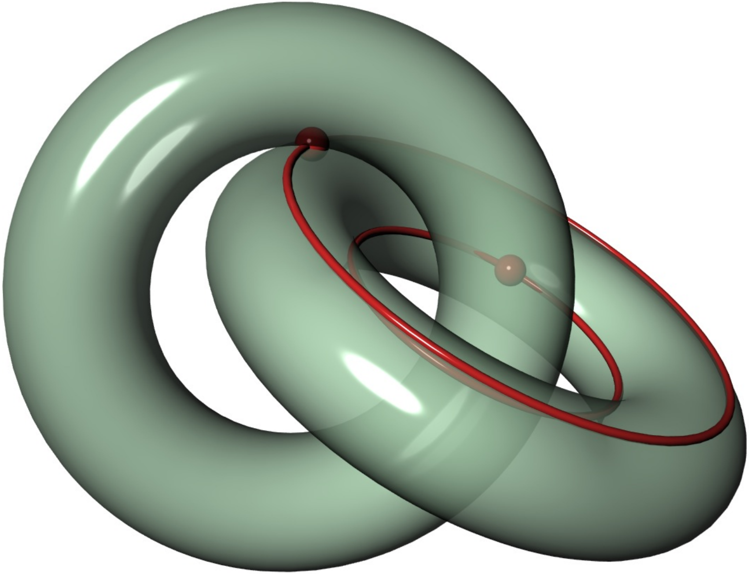

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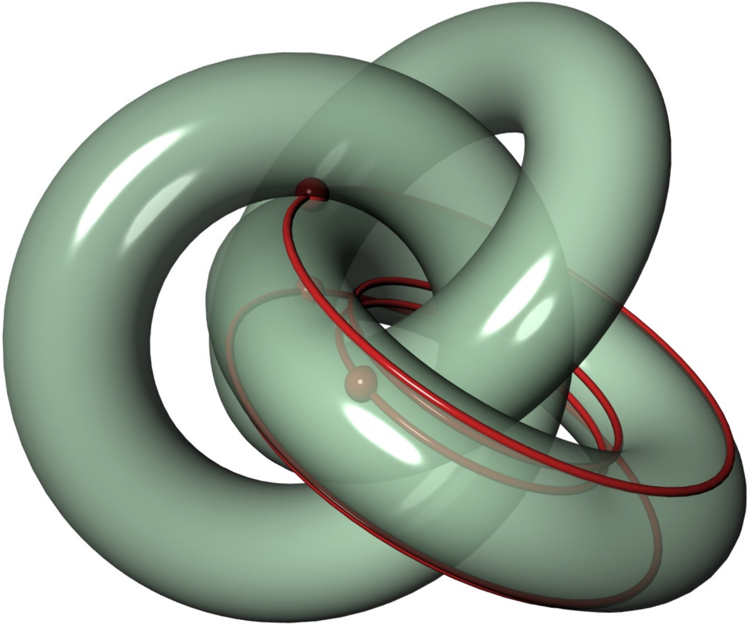

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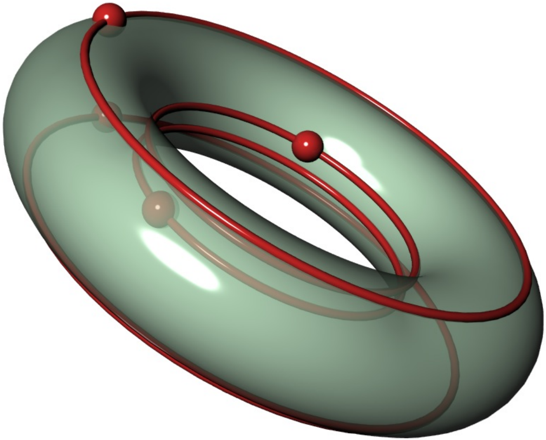

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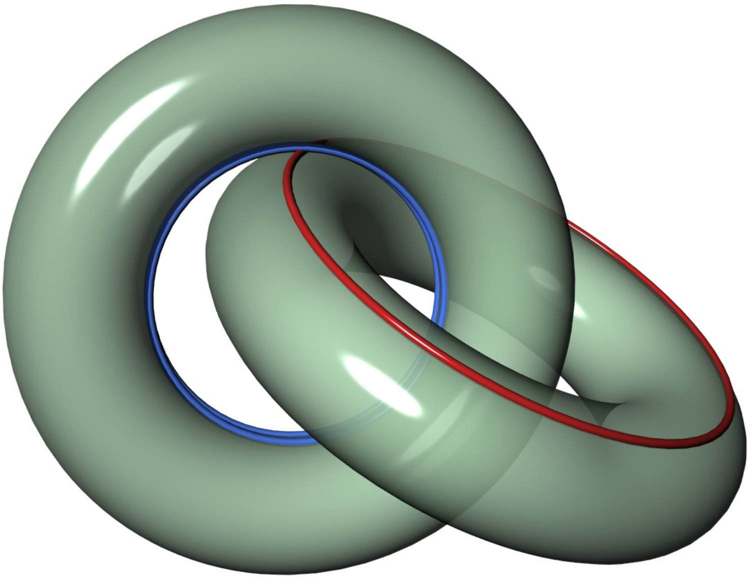
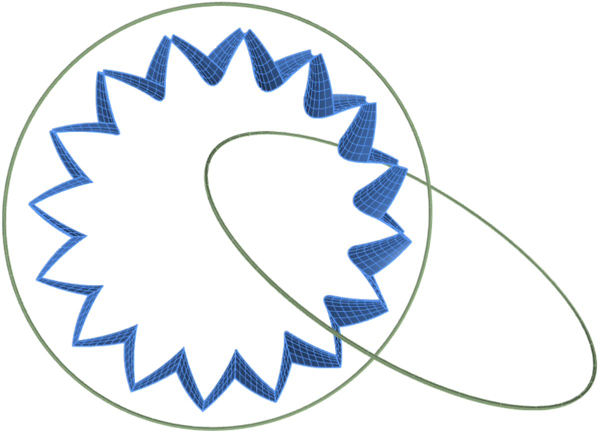


The “inner” teeth are the images of planes in toroidal coordinates.



The “inner” teeth are the images of planes in toroidal coordinates.
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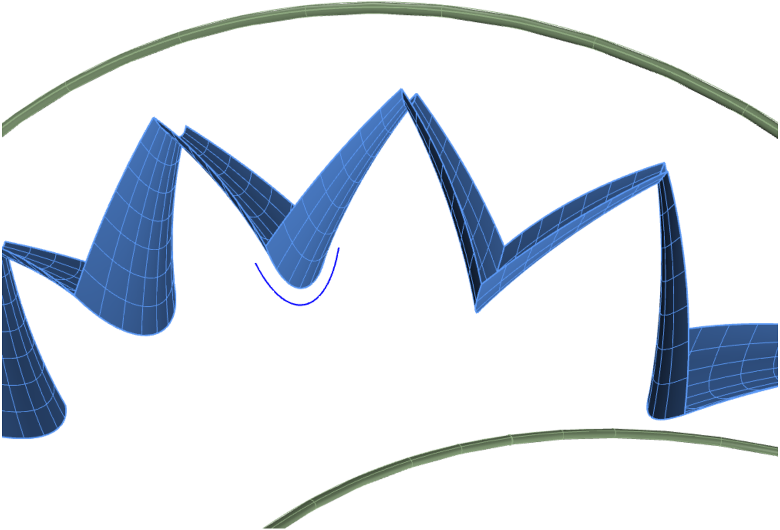

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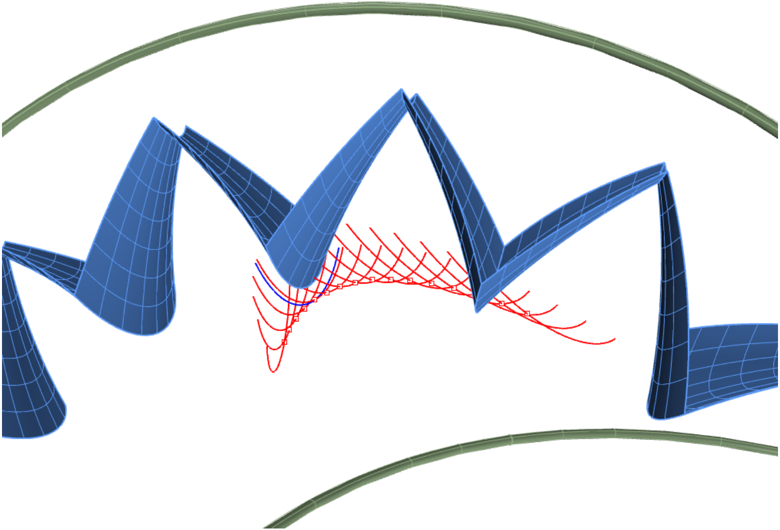

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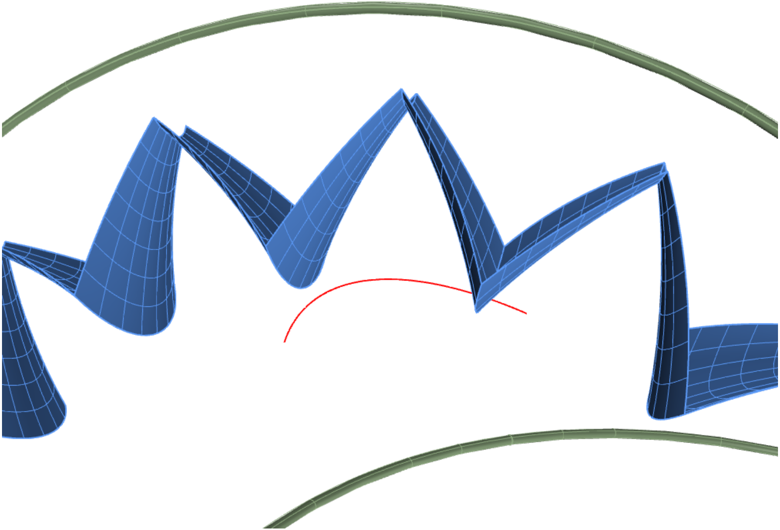

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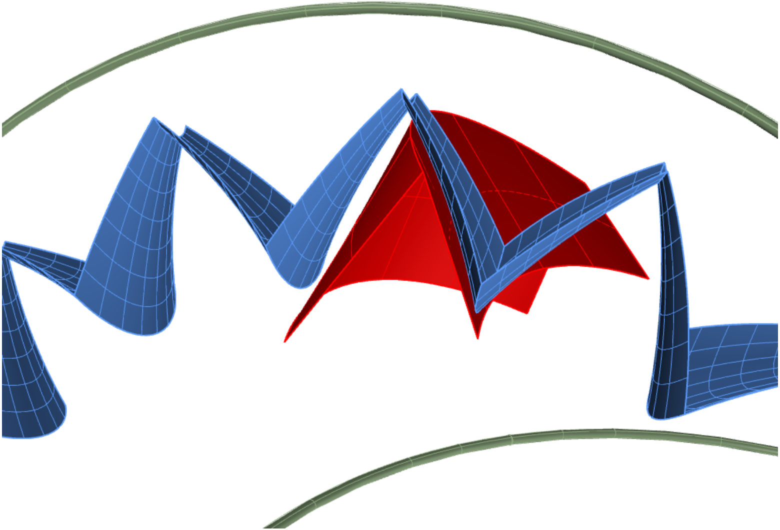

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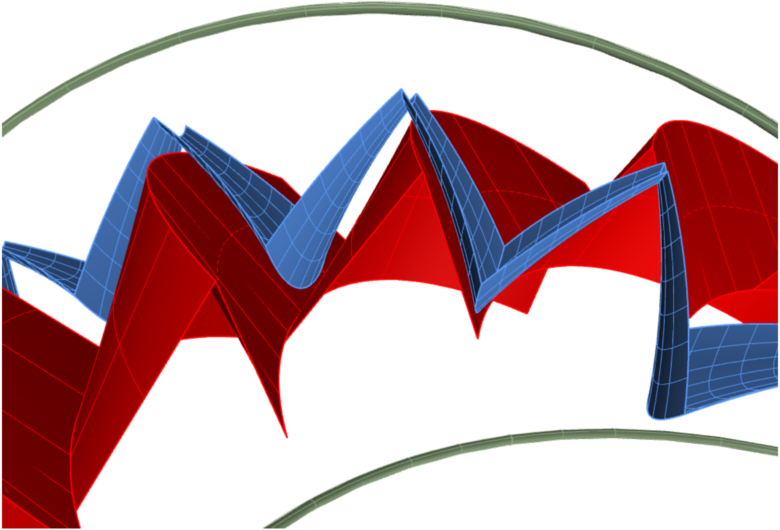

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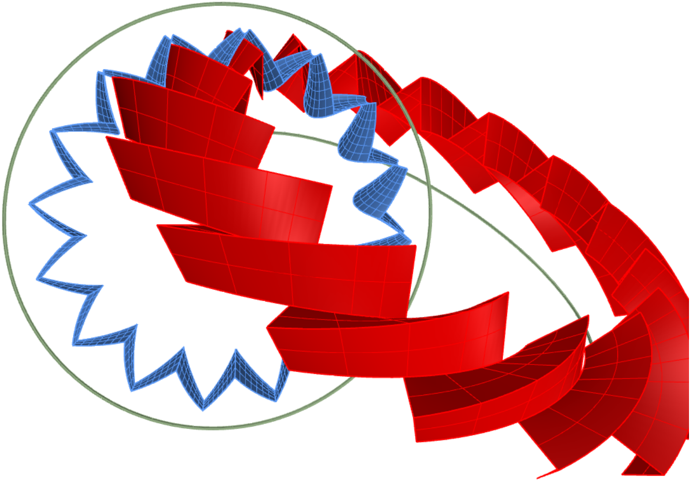
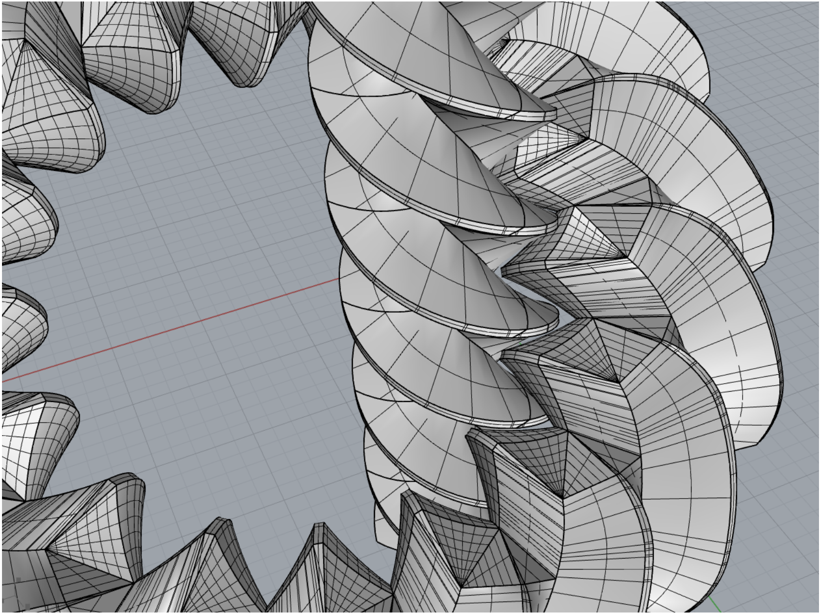
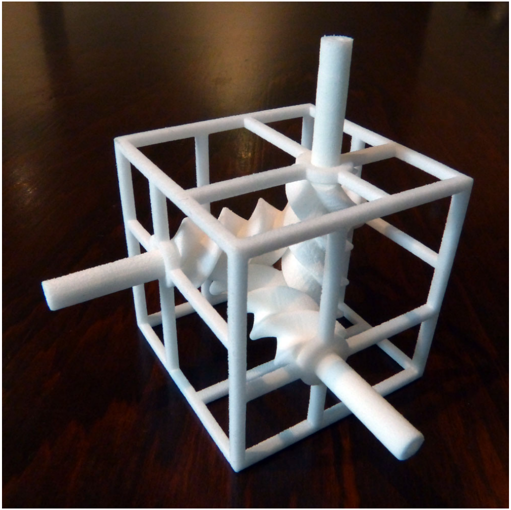


The gears can be powered by a central helical axle.



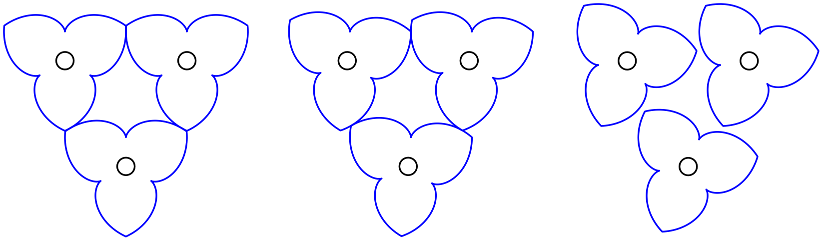
The axle is connected to a motor in the base. Thanks to Adrian Goldwaser for initial prototyping, and to Stuart Young for much more prototyping and construction of the base.

Alternate non-frozen arrangements of three gears



Three helical gears can also pairwise mesh, and they can all move.

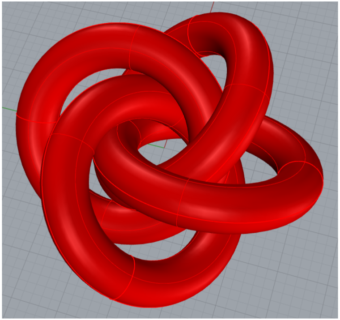
Alternate non-frozen arrangements of three gears



It can even be done with gears with parallel axes!

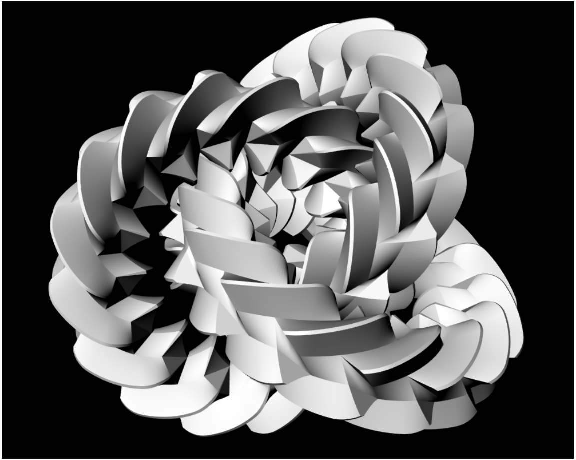
Future directions

- Do the same with the 4-component Hopf link.
- Other configurations of rings?



More generally, we are exploring mechanisms that move in unusual ways.

Thanks!



http://segerman.org

http://ms.unimelb.edu.au/~segerman/

http://youtube.com/user/henryseg

http://shapeways.com/shops/henryseg

http://www.thingiverse.com/henryseg