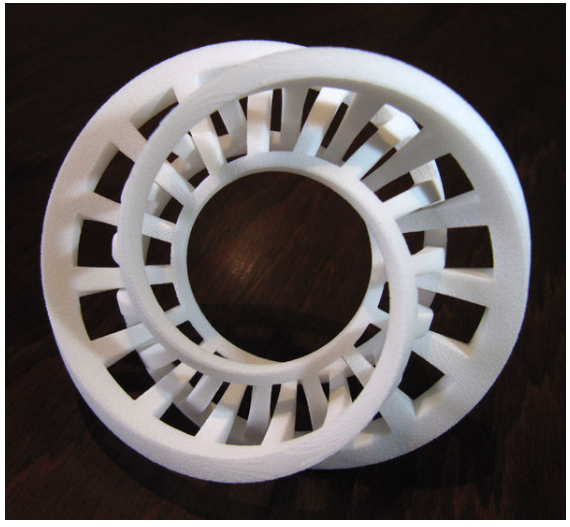
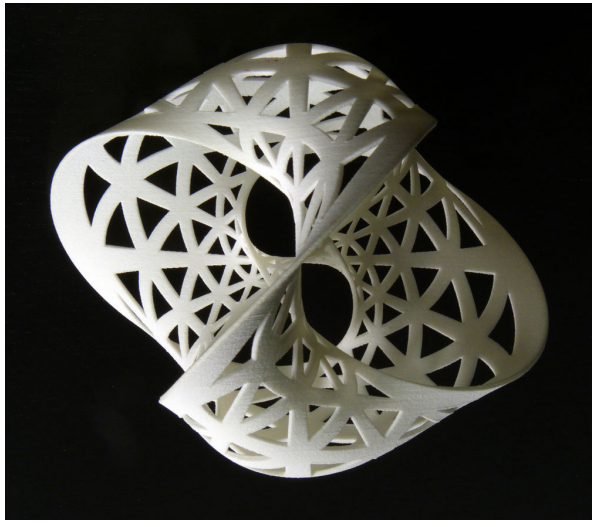
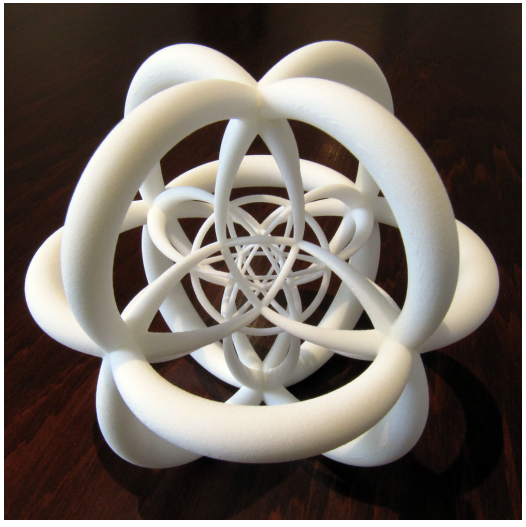




Henry Segerman
Oklahoma State University

3D printing in Mathematics



Part 1:
3D printing technology and how to get into 3D printing

3D printing technologies

Fused deposition modelling

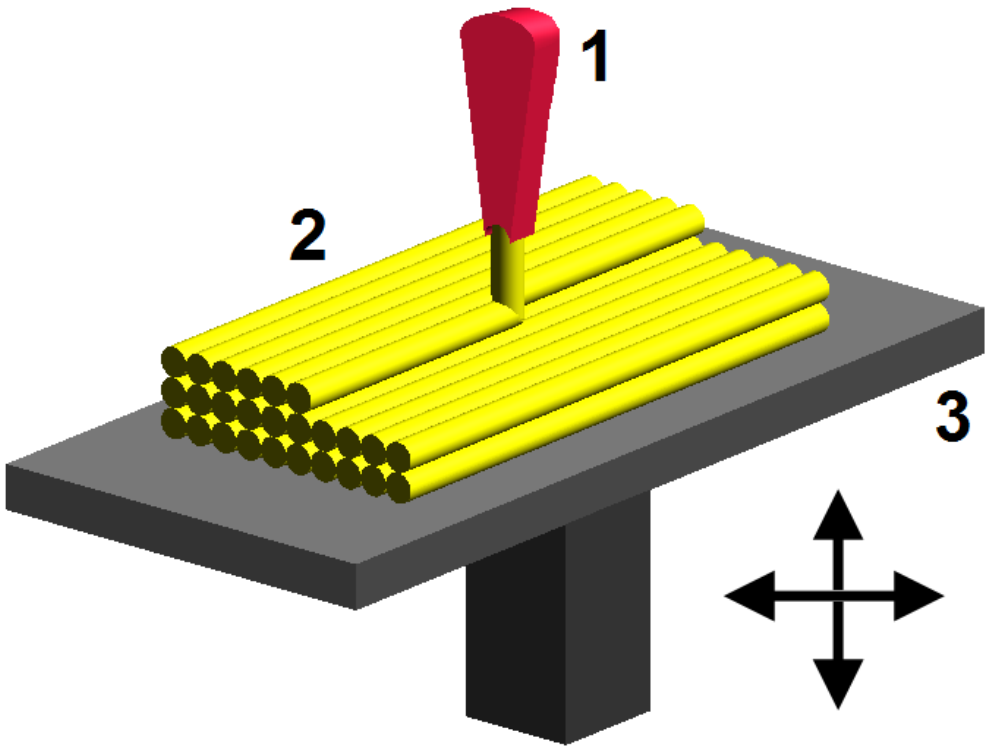


Image credit: Wikipedia - Zureks

3D printing technologies

Stereolithography

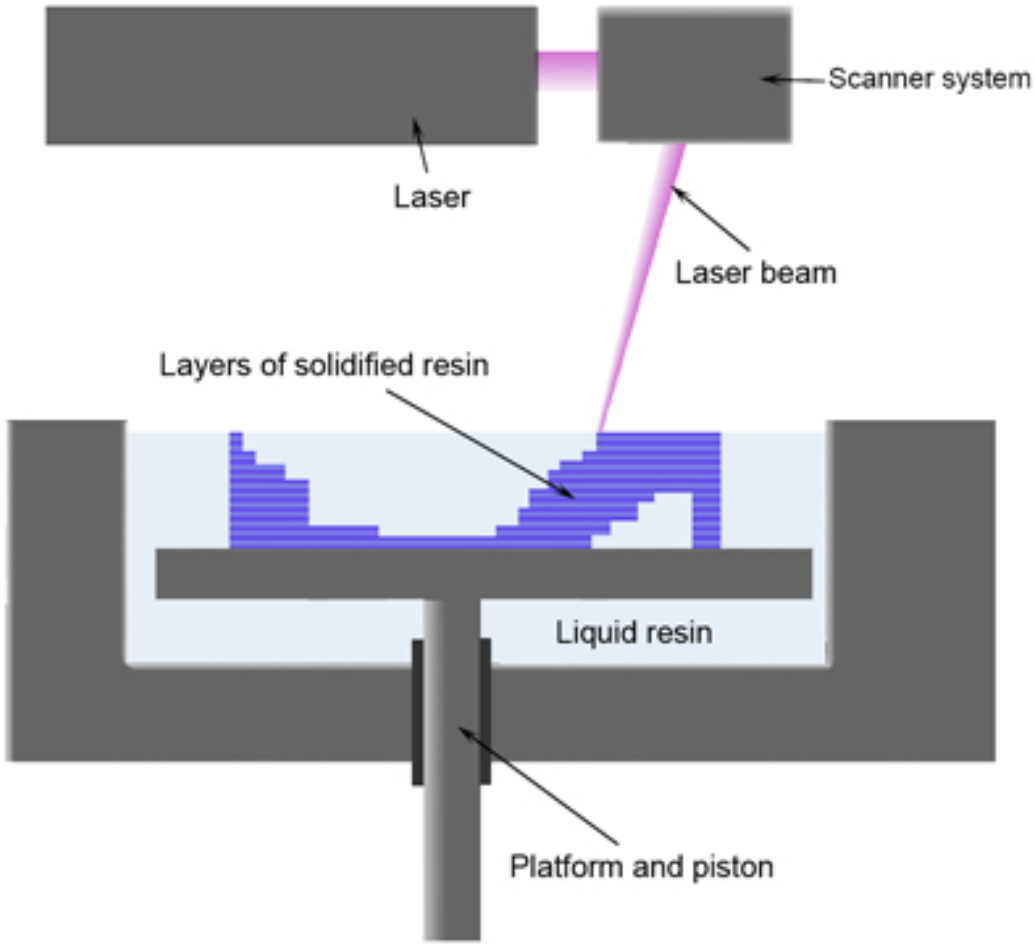


Image credit: Wikipedia - Materialgeeza

3D printing technologies

Selective laser melting

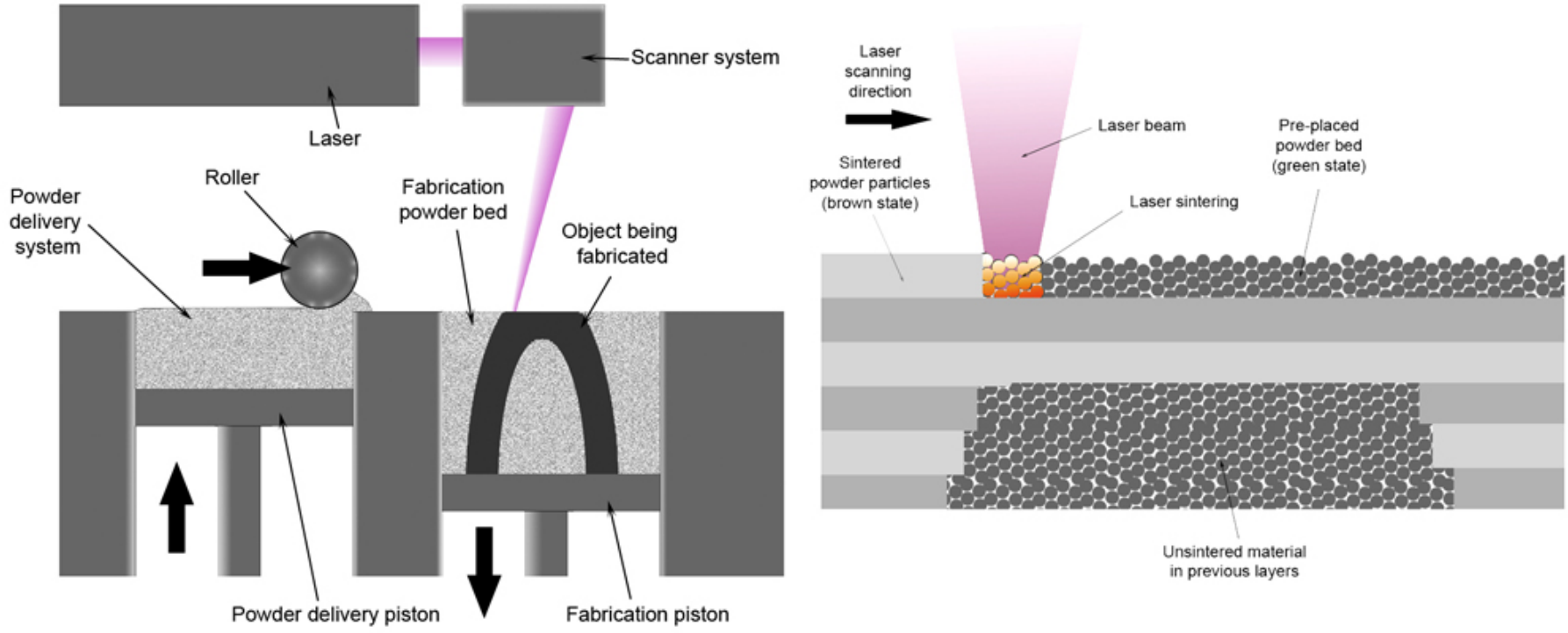


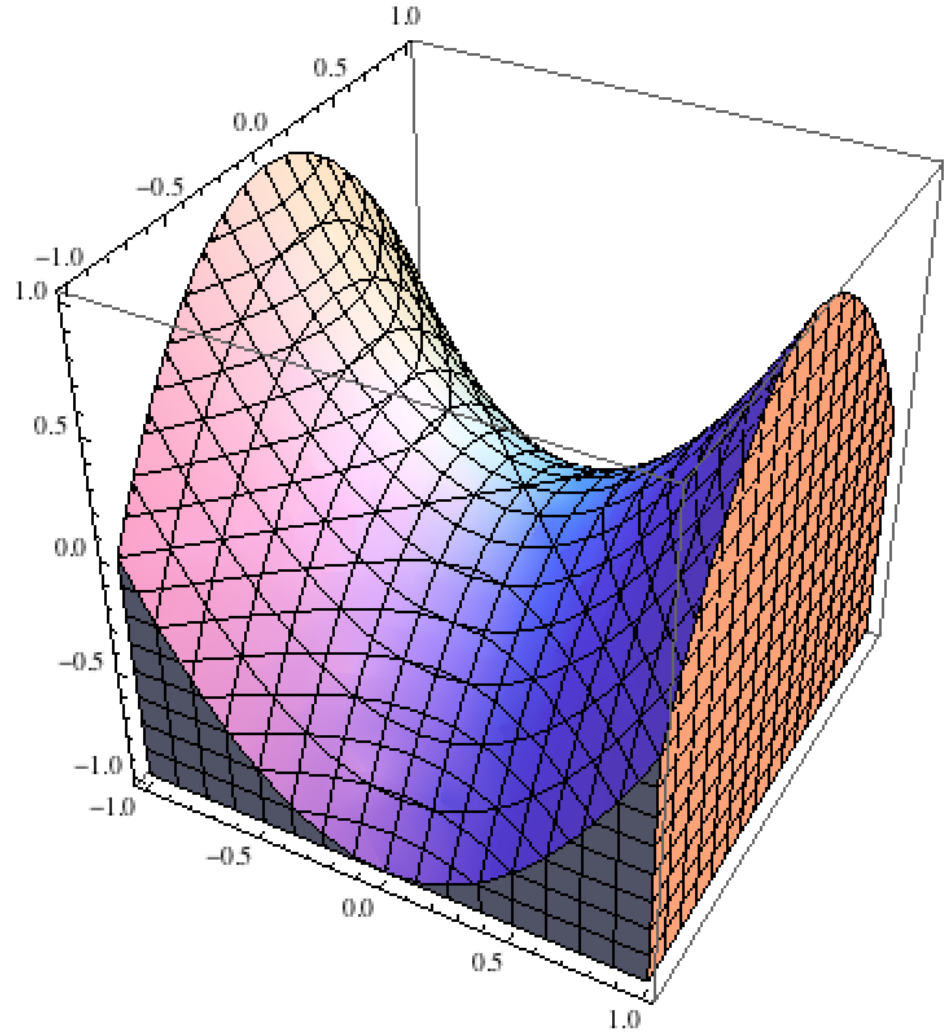
Image credit: Wikipedia - Materialgeeza

How to make models for 3D printing

Mathematica

```
In[9]:= out = RegionPlot3D[z < x^2 - y^2, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}]
        Export["hyperbolic_paraboloid.stl", out]
```

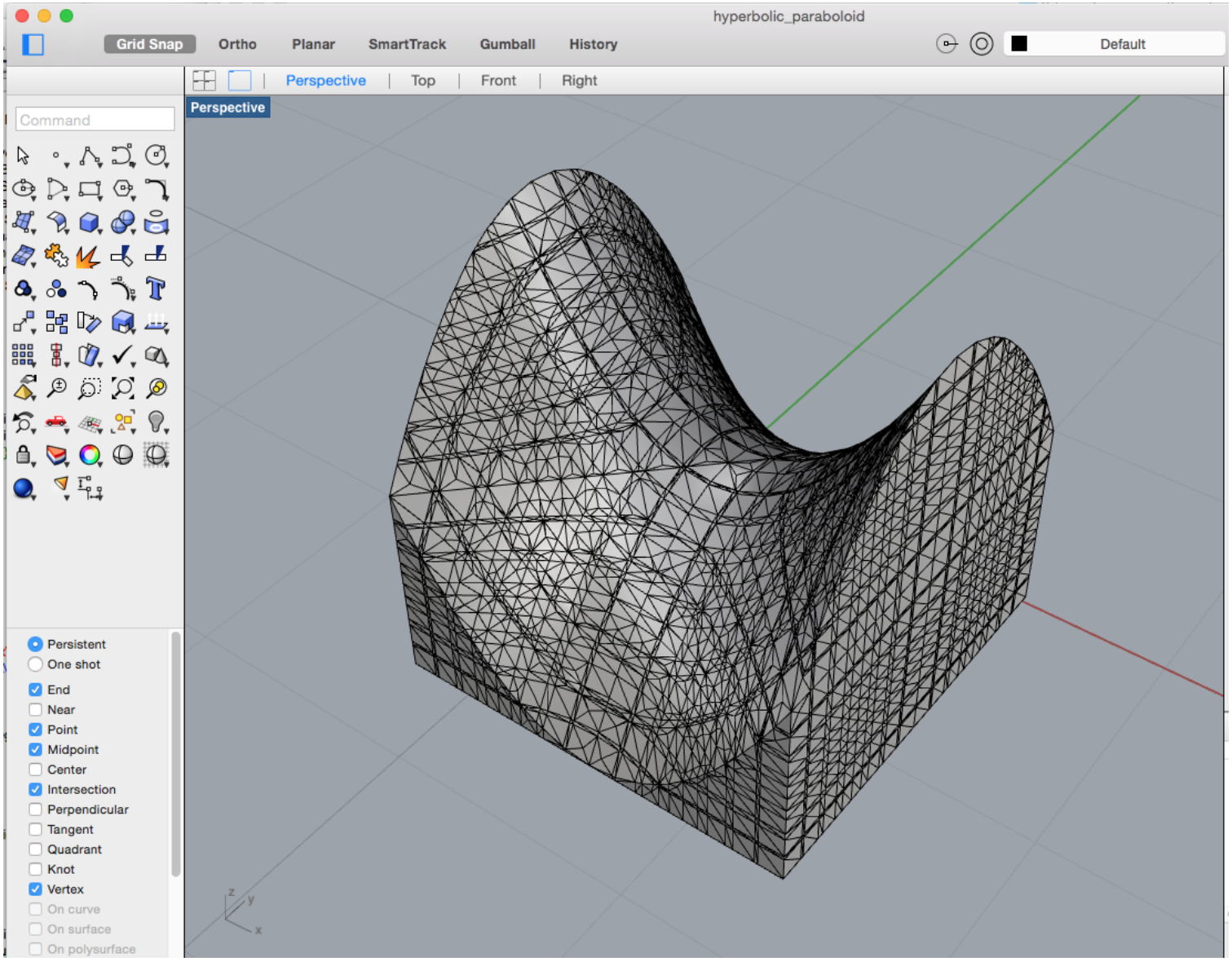
Out[9]=




```
Out[10]= hyperbolic_paraboloid.stl
```

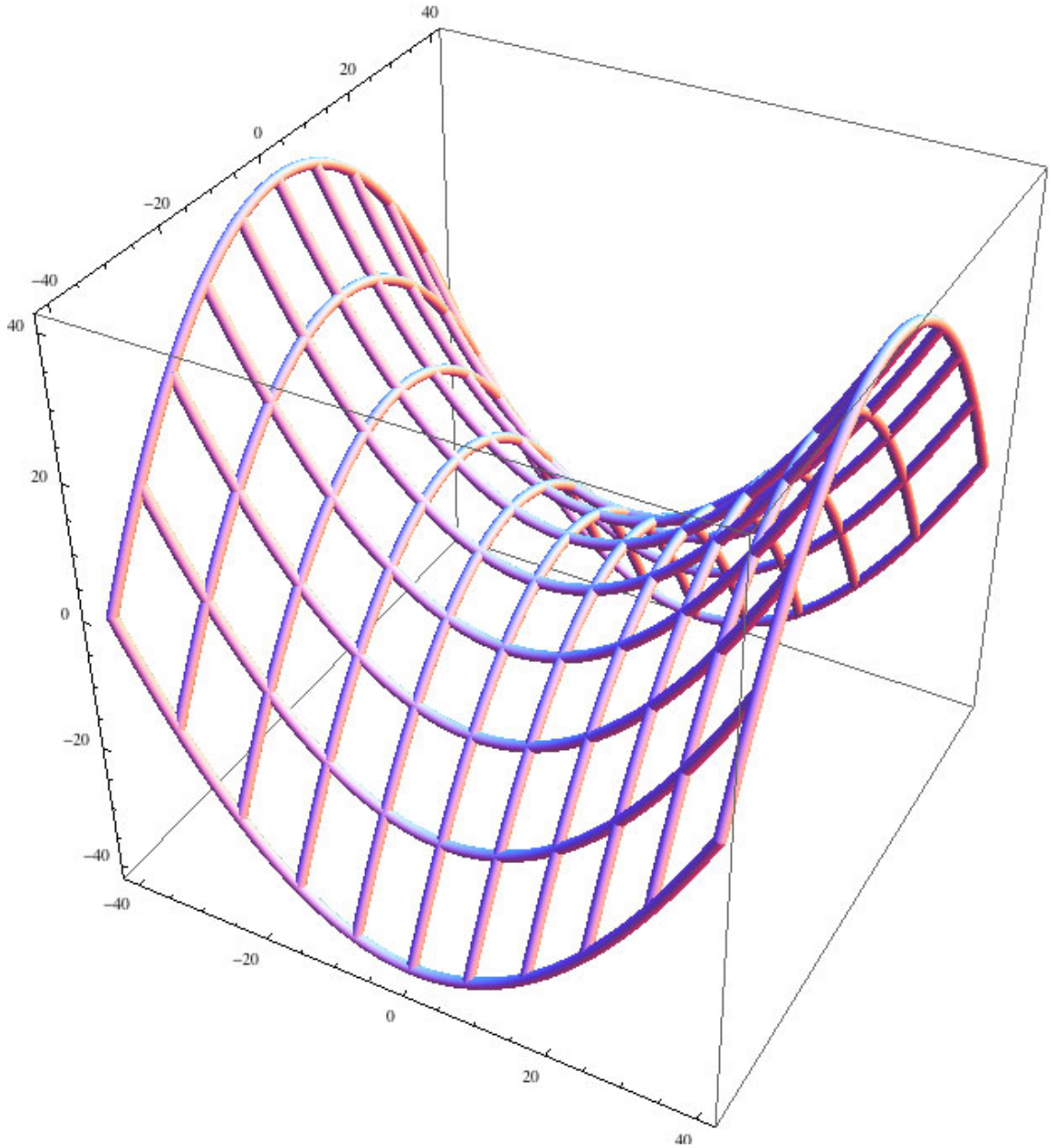
How to make models for 3D printing

Rhinoceros



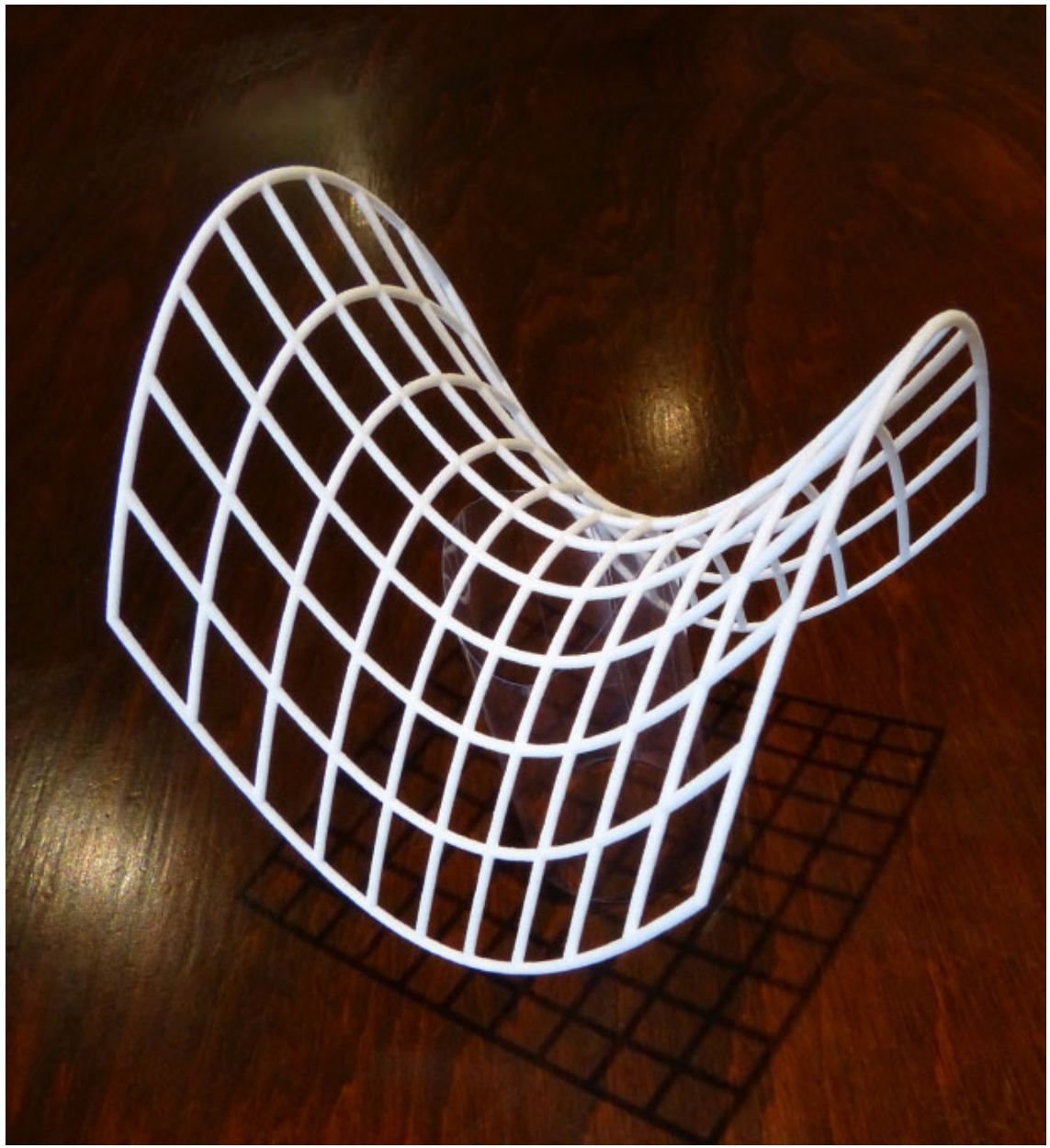
How to make models for 3D printing

Mathematica

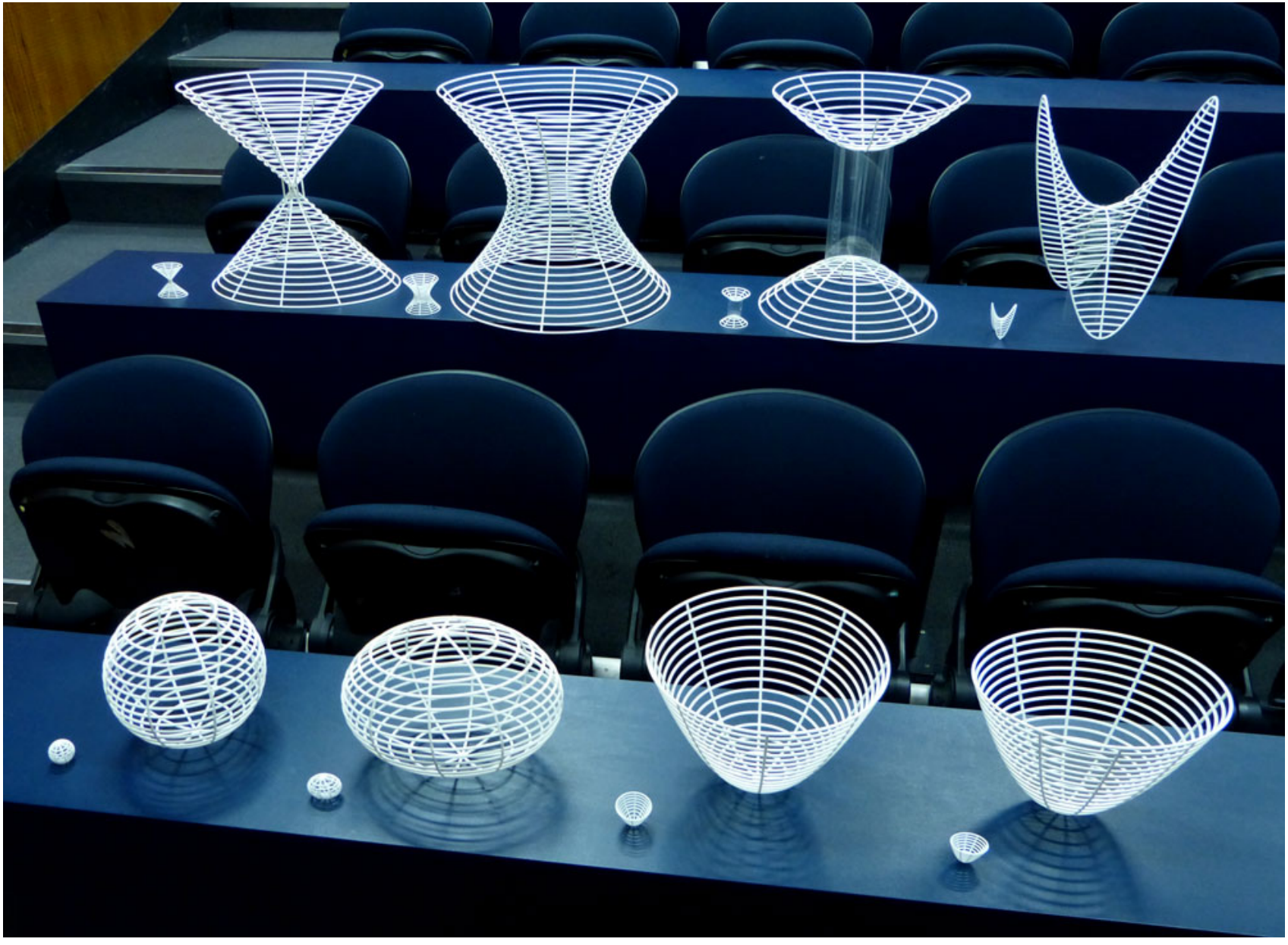


How to make models for 3D printing

Mathematica



Quadric surfaces



Quadric surfaces

Software

Software	Cost	Interface	Notes
Mathematica	Expensive	Text	Huge amounts of built-in math
OpenSCAD	Free	Text	Constructive solid geometry
Tinkercad	Free	Graphical	Easy to get into
Blender	Free	Graphical	Powerful, scary user interface
Rhinoceros	Not cheap	Graphical	Best for mathematical designs
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etc.			

(Rhinoceros demo)

Part 2:
3D printing in education

Two main uses

1. Make manipulatives and visualizations
    - E.g. quadric surfaces, many more examples in Part 3.

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- Do we need a 3D printer ourselves? Either:
  - Buy own.
    - Requires money, space, maintenance... but fast(ish) turnaround time.
  - Use an online service (e.g. `Shapeways.com`).
    - No startup cost but more expensive per print, higher quality, more design freedom.

Why have students learn 3D printing?

- Highly motivating - students get to take home something they made with their mind.
- Consolidates students' math skills - use on real world problems.
- 3D printing is perfect for project work, difficult to do in math otherwise.
- Students self-correct errors: they see that their solution doesn't work. (Compare with exams - incorrect answer "doesn't matter" and they don't find out until next week.)
- 3D technology is likely to be highly relevant for future careers.

"The more math you know, the more stuff you can make"
- George Hart

MoMath - Mathenaeum

MoMath - Mathenaeum

Use in High School - UK Department of Education report

https://www.gov.uk/government/publications/3d-printers-in-schools-uses-in-the-curriculum

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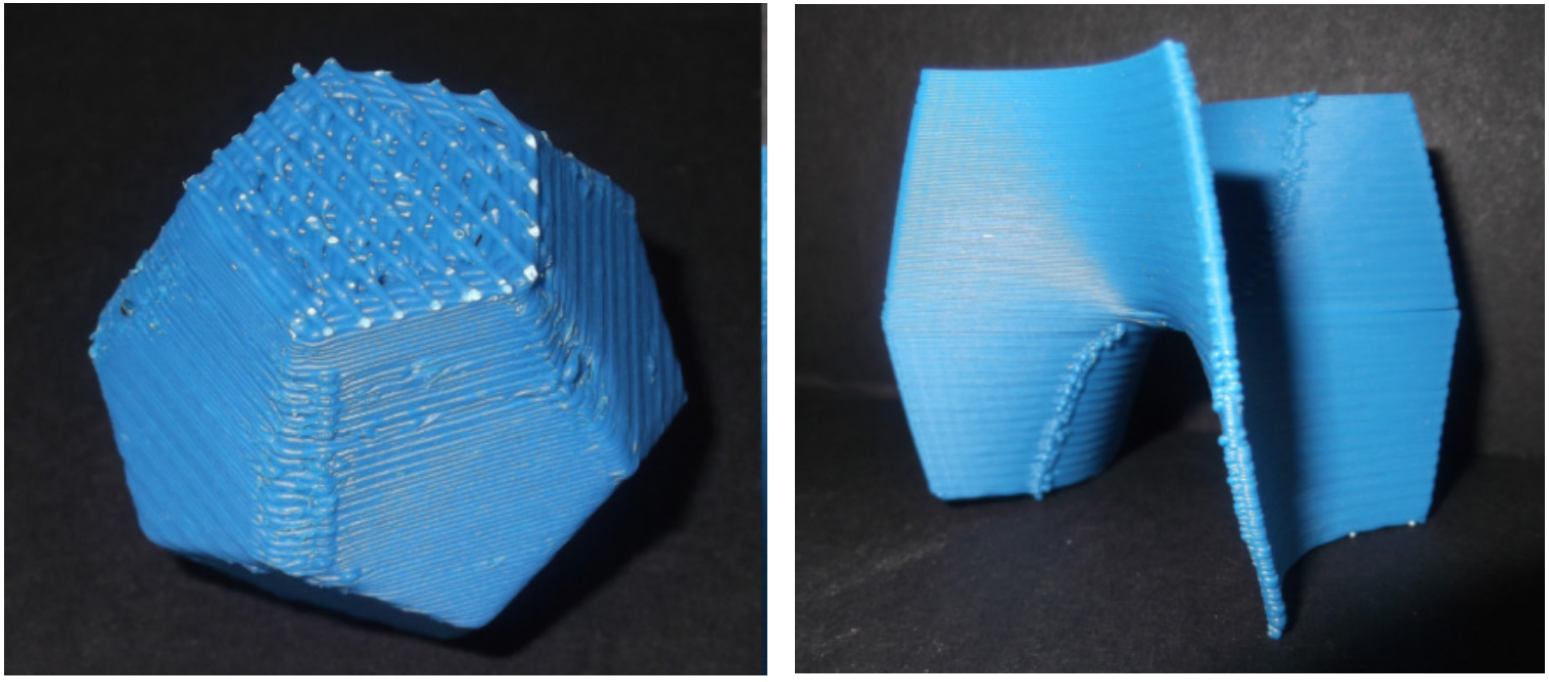

Figure 8 - At Watford Grammar School for Boys the printer was used to demonstrate a 3D graph for various algebraic equations as well as producing examples of regular shapes (Dodecahedron).

https://www.gov.uk/government/publications/3d-printers-in-schools-uses-in-the-curriculum

Use in High School - UK Department of Education report

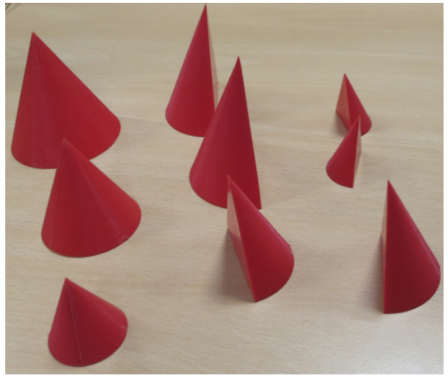

Figure 10 - The Kings School, Peterborough made a set of cones and part cones for a mathematics investigation into areas and volumes.

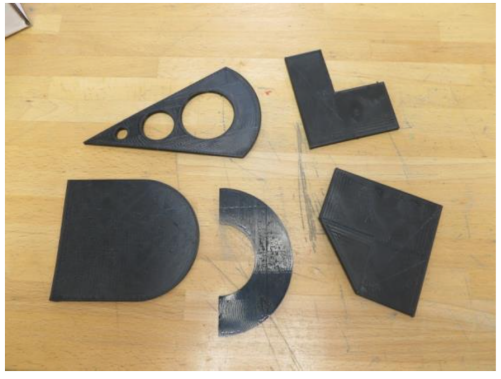

Figure 11 - At Highworth Grammar School for Girls, a set of laminae were made to support the teaching of centres of mass in mechanics instruction.

https://www.gov.uk/government/publications/3d-printers-in-schools-uses-in-the-curriculum

Use in High School - UK Department of Education report

Excerpts

- "Equipping pupils to understand the application and potential of this new type of technology will be important to helping prepare them for a world in which similar technologies will be increasingly commonplace, particularly in STEM contexts."
- "The 3D printer is ideally suited to project work, where learning arises naturally as part of an investigation or construction project."
- "...the printers had a highly motivational effect on pupils and most schools reported a greater interest in STEM subjects."

`https://www.gov.uk/government/publications/3d-printers-in-schools-uses-in-the-curriculum`

Use in College - Laura Taalman, JMU

MATH 297: Current Knot Theory Research and 3D Printing Policy and Syllabus

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Instructor:	Laura Taalman, Professor in the Department of Mathematics and Statistics
Email:	taalmala@jmu.edu
Website:	educ.jmu.edu/~taalmala
Class site:	www.geekhaus.com/3space
Meetings:	Wednesdays 2:30pm-4:30pm in the JMU 3-SPACE classroom (Burruss 349)
Textbook:	The internet!

`http://geekhaus.com/3space`

Use in College - Laura Taalman, JMU

Course Objectives

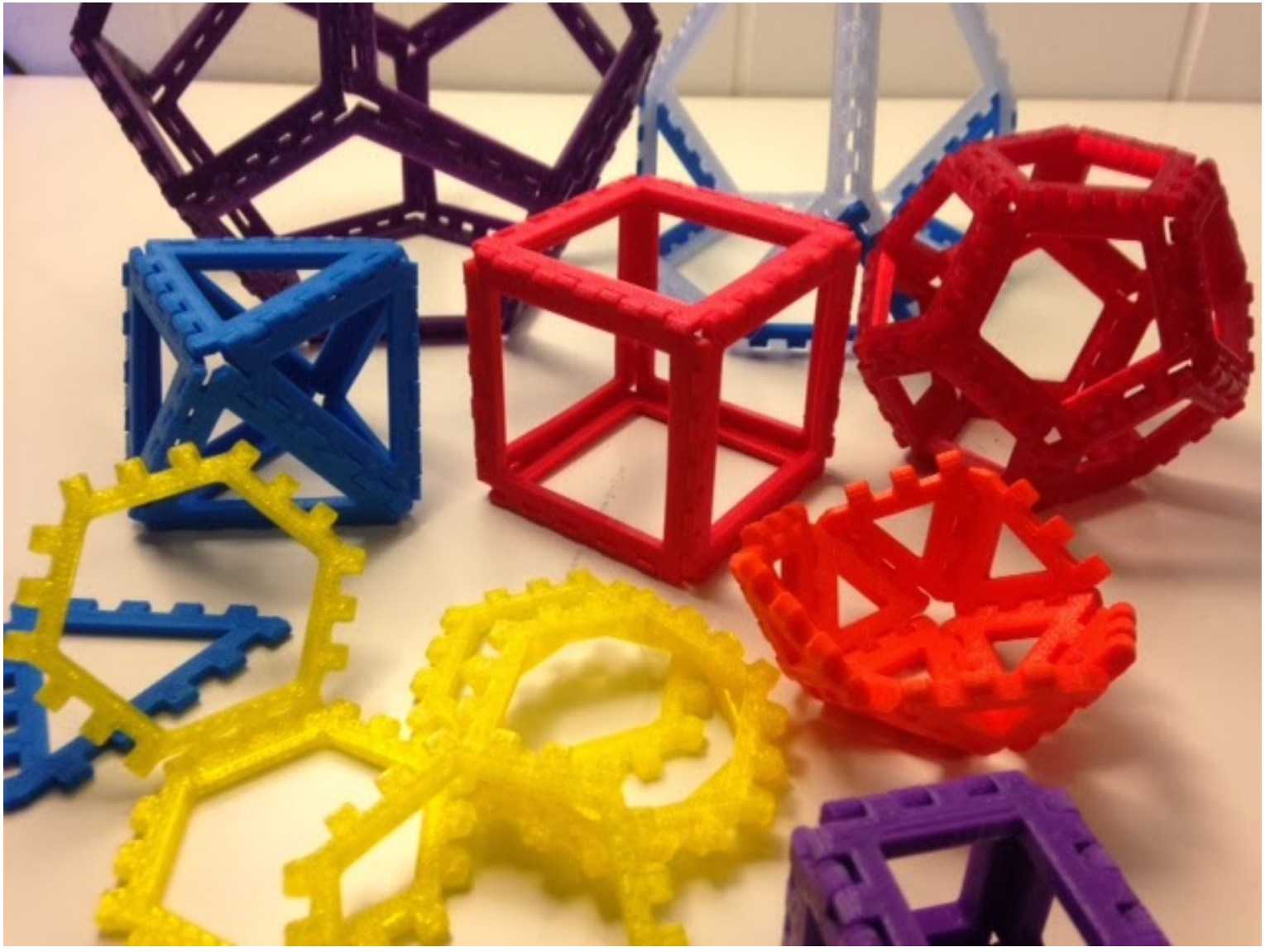
1. Study current accessible research in knot theory and attempt to come up with new results and/or generate and examine examples that support the current literature.
2. Use 3D printers to produce actual physical models of the knots and related objects that are in the papers you are reading. You may have to learn Mathematica, Maple, MATLAB, OpenSCAD, TopMod, or other programs to help you create the models.
3. Learn how communicate mathematics through written papers, blog posts, and presentations using LaTeX, Beamer, WordPress, Blogger, and other typesetting and presentation software.
4. Complete applications for REU programs and other future research projects.
5. Present your work as a talk and as a poster at the MD/DC/VA MAA spring that will be held at JMU this spring on April 25-26.
6. Create and print a final project that will be housed in the front foyer display case in the Department of Mathematics and Statistics in Roop Hall.

`http://geekhaus.com/3space`

Use in College - Laura Taalman, JMU

http://geekhaus.com/3space

More from Laura Taalman



More from Laura Taalman

More from Laura Taalman

Day 365!!! Last day of the print-every-day-for-a-year project!

I think the first thing that needs saying here is that a year is a very long time. There are a lot of days in a year. After about 60 or 70 days I remember thinking that we were probably halfway through this year-long blog thing and then being shocked at just how many days there are in a year. Damn.

But it is over, we have finished the year and so we get a TROPHY. Since our 3D-printing journey began with the desire to print knots, our trophy is a tiny 3D-printer (model from RichRap's very cleverly designed Advent Makerbot Replicator 2), which is printing an even tinier knot (model dating back all the way to the beginning of this journey, on Day 9):

http://makerhome.blogspot.com

More from Laura Taalman

HACKTASTIC

PolyBowls – From zero to OpenSCAD in 6 minutes

mathgrrl August 28, 2015 0 Comments

The following collection of bowls and pen holders we are all generated from the same simple OpenSCAD code:

ABOUT HACKTASTIC

This is a blog about design, math, and failure by @mathgrrl.

MATHGRRL DESIGNS

Customize and download hundreds of 3D models at the mathgrrl page on Thingiverse.

WISDOM COLLECTORS

New to 3D design? Me too. These Wisdom Collectors gather notes and resources for getting started with three.js, Maya, OpenSCAD, and the math of meshes.

http://mathgrrl.com/hacktastic/

Use in College - Chris Hanusa, Queens College, CUNY

Math with Mathematica

Welcome to Math 213, Math with Mathematica, this Spring 2015!

Virtual Art Gallery

Course Information

- Course Syllabus
- Course Calendar
- Course Content, including a list of topics covered, in-class tutorials, and homework assignments.
- Information about the Course Projects.
- Information about Mathematica Access (On MyQC; login required)
- A link to Google Classroom

Key Dates: (subject to change)

- **First Day of Class:** Wednesday, January 28
- **Quiz 1:** Wednesday, February 11
- **Project 1 Due:** Wednesday, March 4
- **Quiz 2:** Monday, March 9
- **Project 2 Due:** Wednesday, April 1
- **Project 3 Due:** Wednesday, May 13
- **Last Day of Class:** Wednesday, May 13
- **Project Presentations:** Wednesday, May 13 and Final Exam Day, Monday, May 18 from 8:30–10:30

http://qc.edu/~chanusa/courses/213/15/index.html

Use in College - Chris Hanusa, Queens College, CUNY

http://qc.edu/~chanusa/courses/213/15/index.html

Use in College - Chris Hanusa, Queens College, CUNY

http://qc.edu/~chanusa/courses/213/15/index.html

Use in College - Dave Bachman, Pitzer College

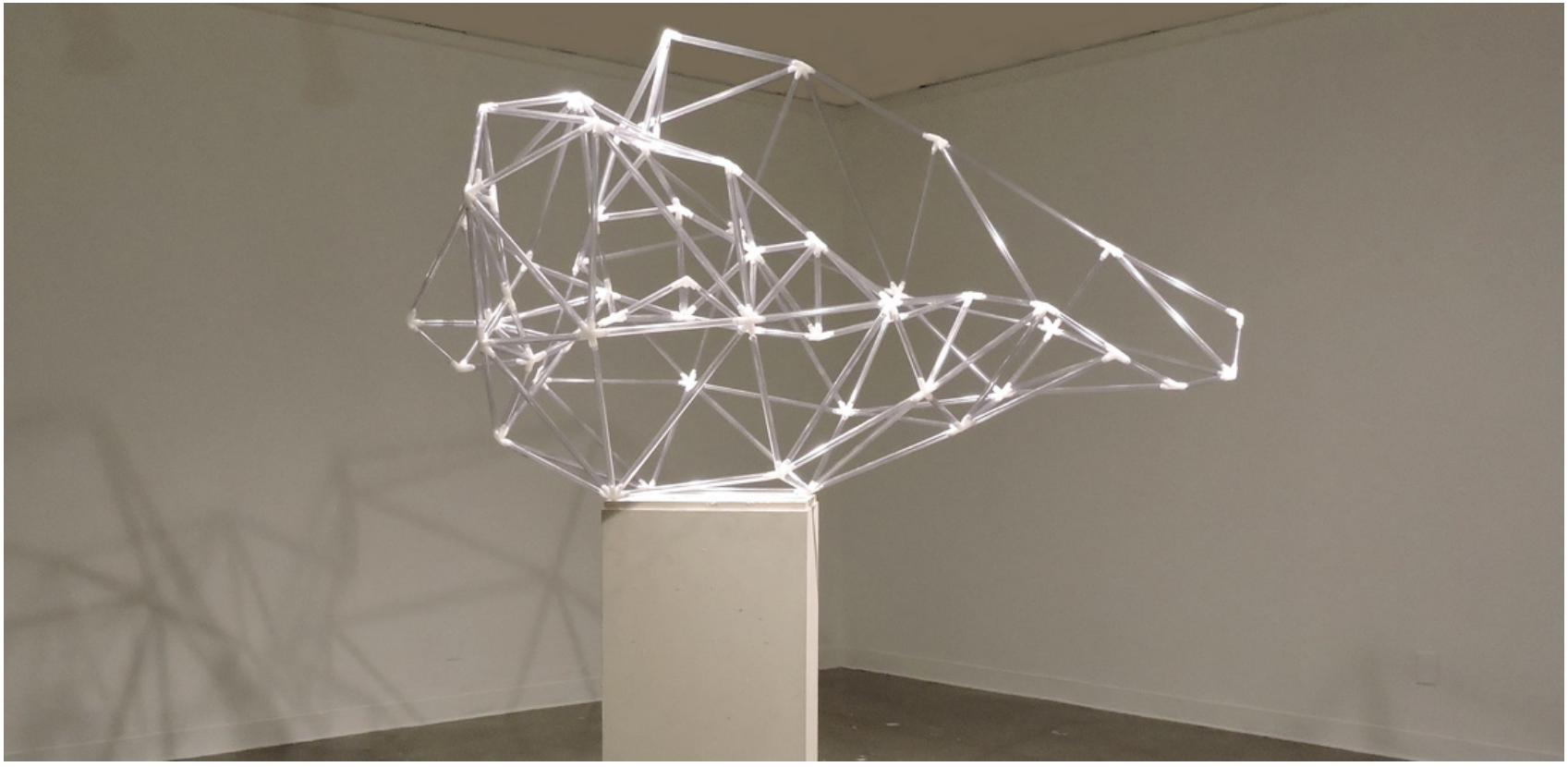
More from Dave Bachman

http://davidbachman.org

More from Dave Bachman

`http://davidbachman.org`

More from Dave Bachman



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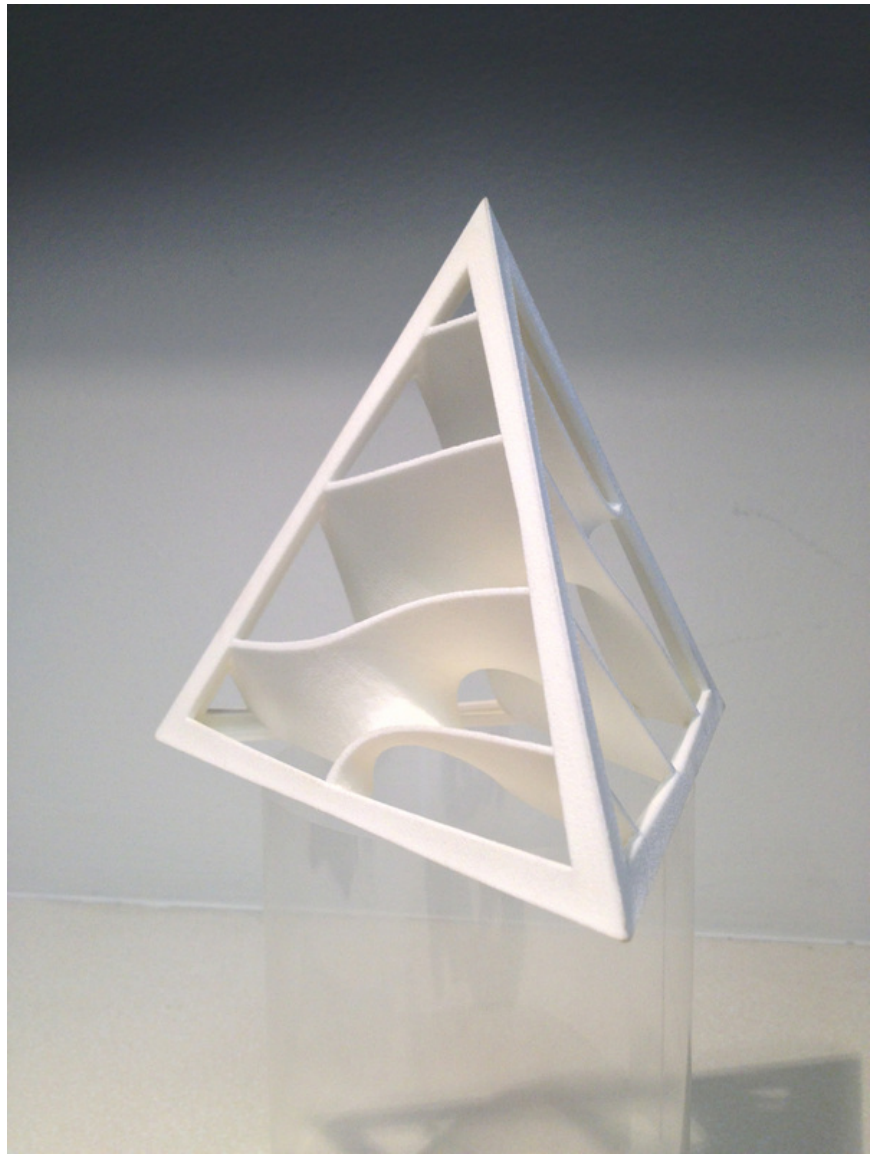
More from Dave Bachman



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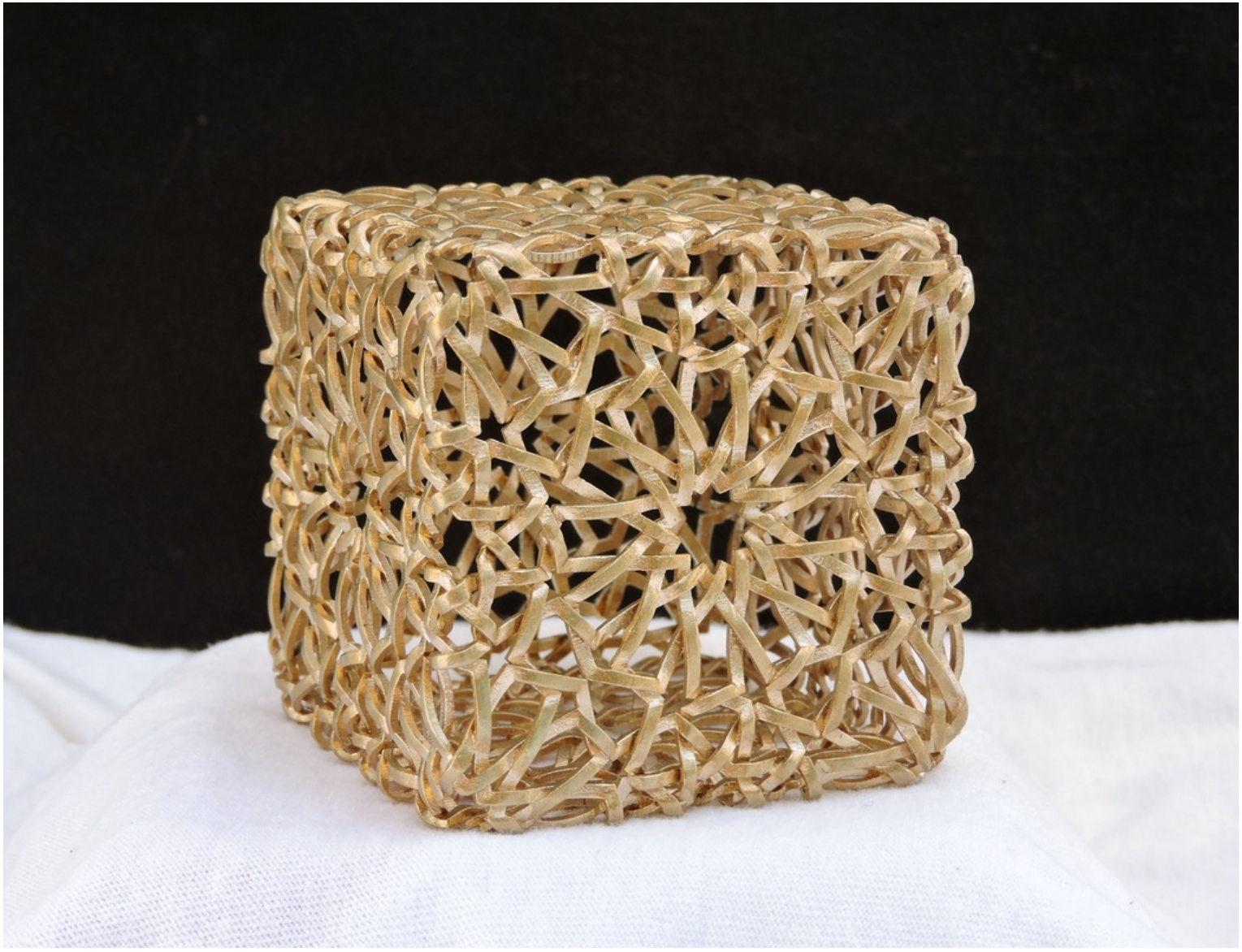

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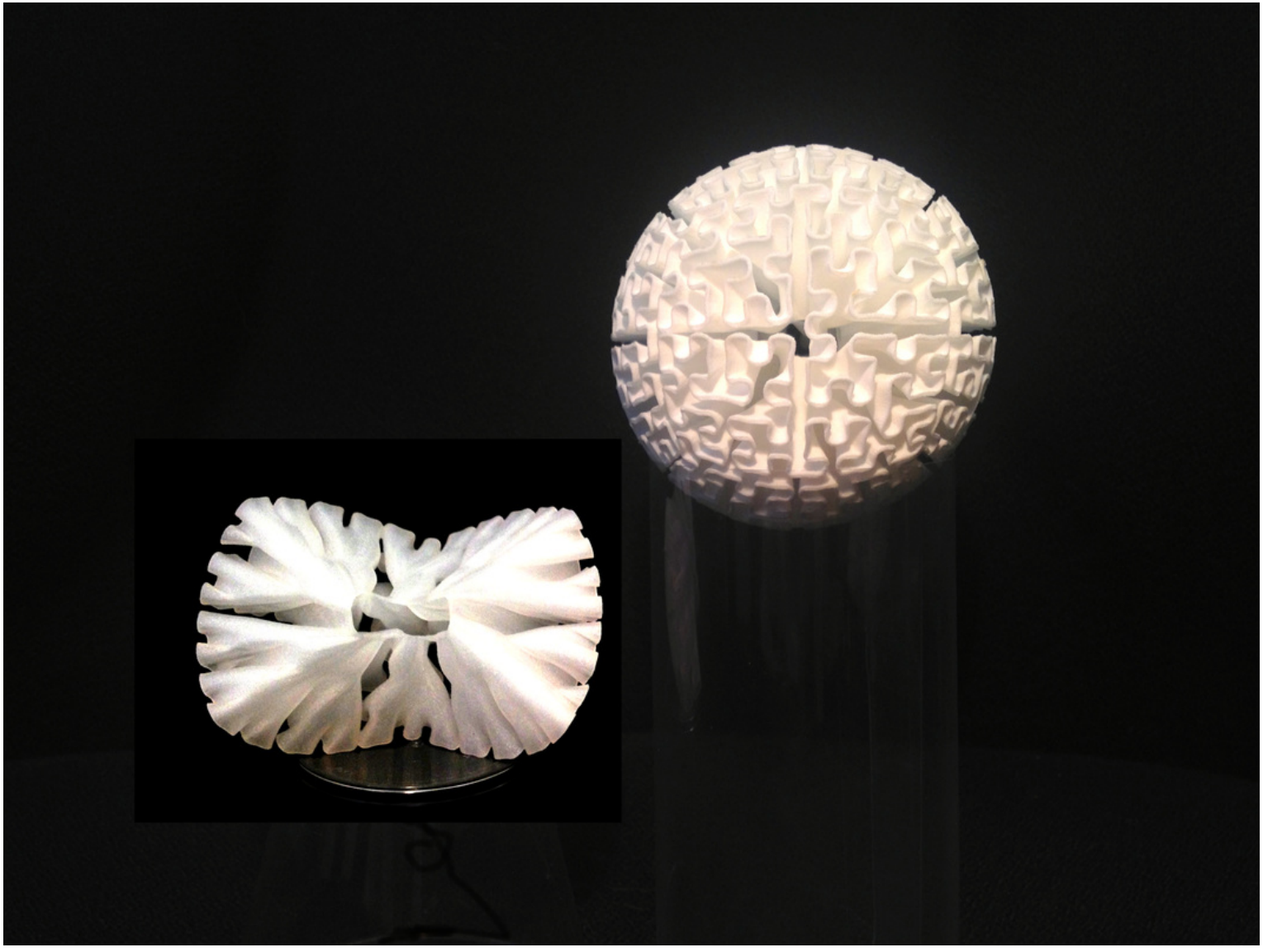


(Joint with Robert Fathauer and me.)

`http://davidbachman.org`

Use in College - me

Math 4423: Geometry and Algorithms in Three-dimensional Modeling

Text: *Applied Geometry for Computer Graphics and CAD* (Springer Undergraduate Mathematics Series) 2nd Edition), by Duncan Marsh. (The text is freely available in electronic form from `library.okstate.edu`.)

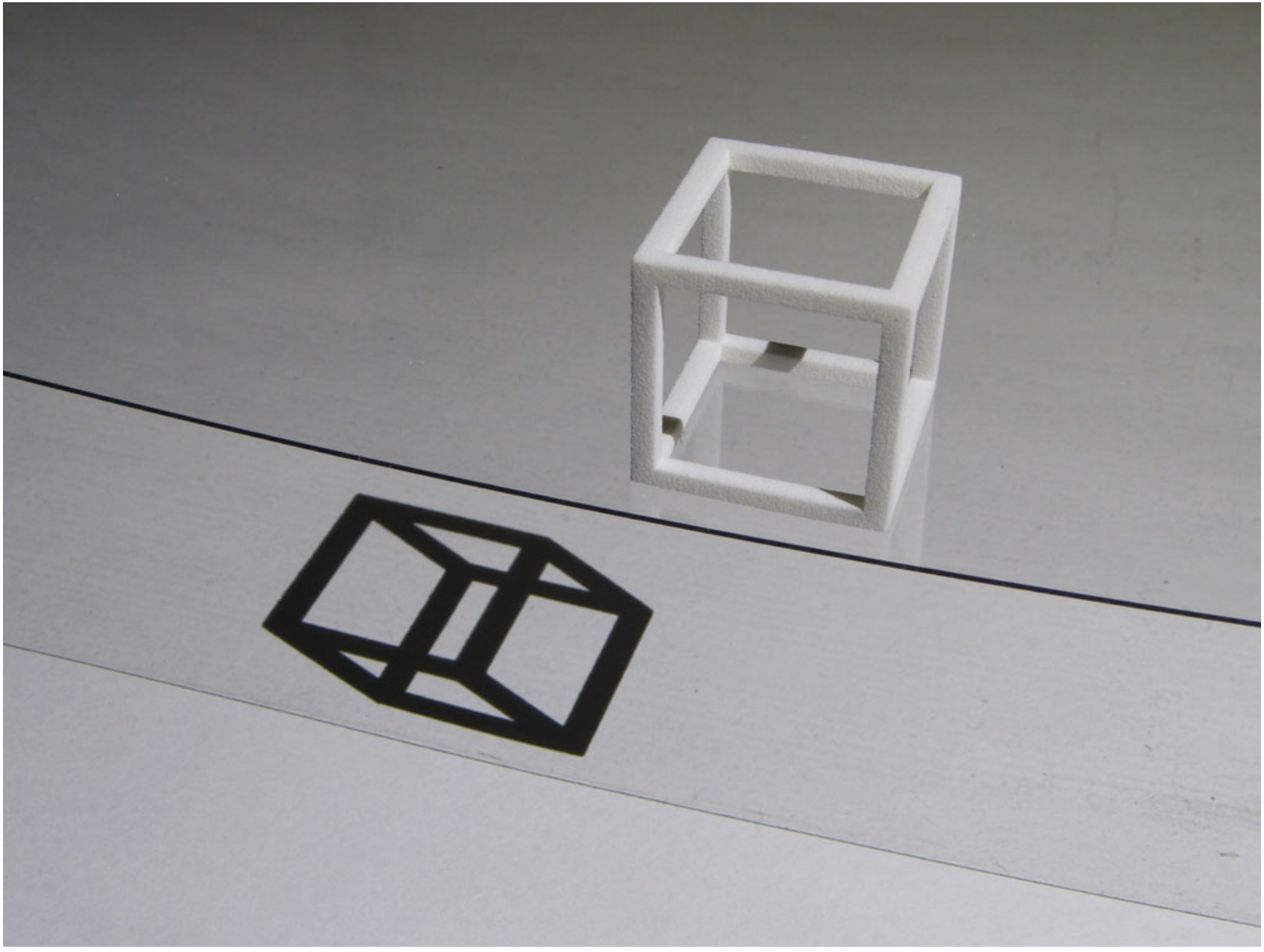
Course Objectives: There are two goals of this course.

- Students will learn some of the mathematical underpinnings behind the ways in which three-dimensional content is represented, generated and modified using computers.
- Students will apply their knowledge of these topics: they will learn how to use the CAD (Computer-Aided Design) program Rhinoceros, including automation with the programming language Python, and Makerbot 3D printers. They will use all of these to design and 3D print models at the 3D printing lab in the Department of Mathematics. Models will illustrate mathematical concepts, and/or be drawn from students' other academic interests. Students will write reports describing the mathematics, design choices, and other relevant details behind their models.

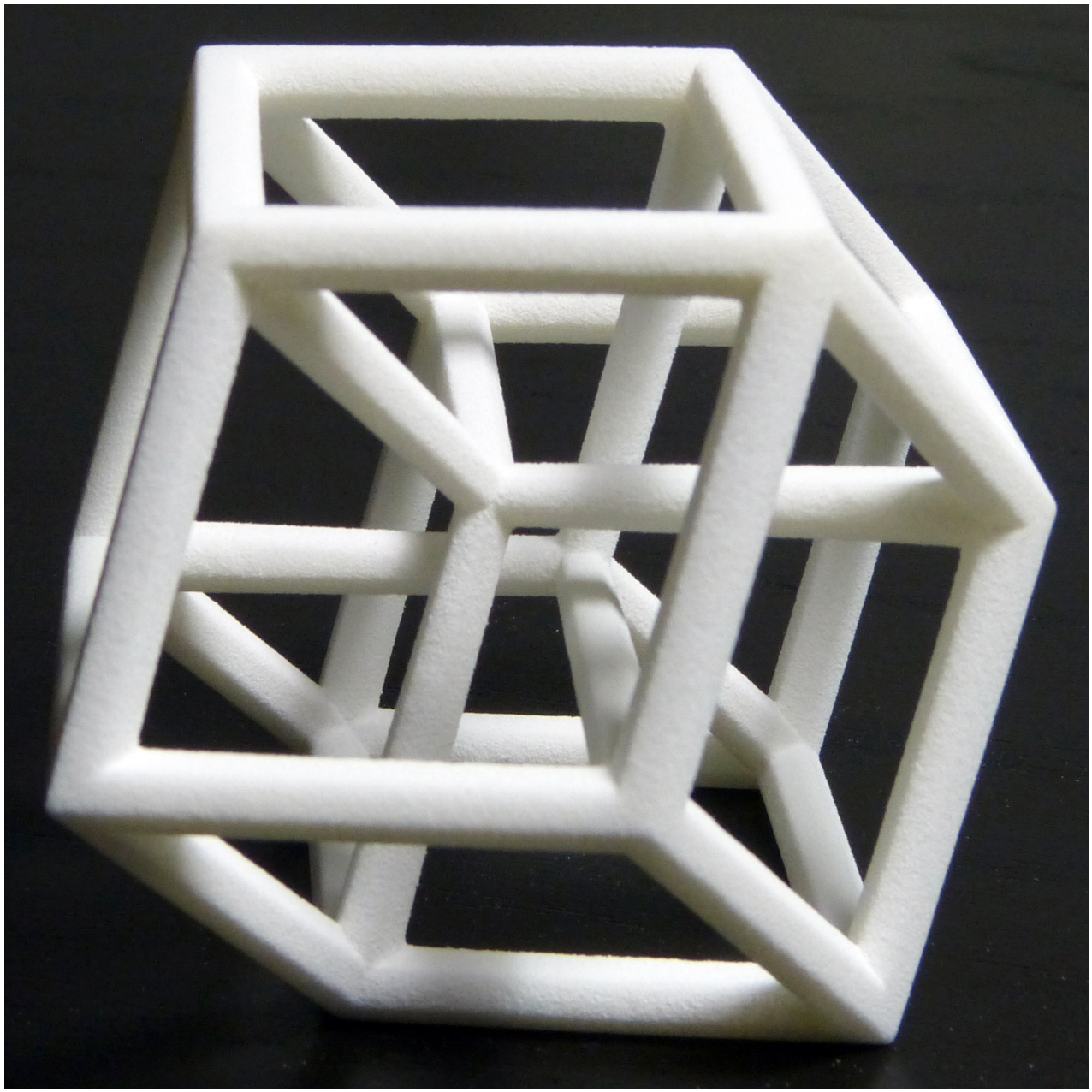
Use in College - me

Part 3:
3D printing for mathematical visualisation

Parallel projection of a cube

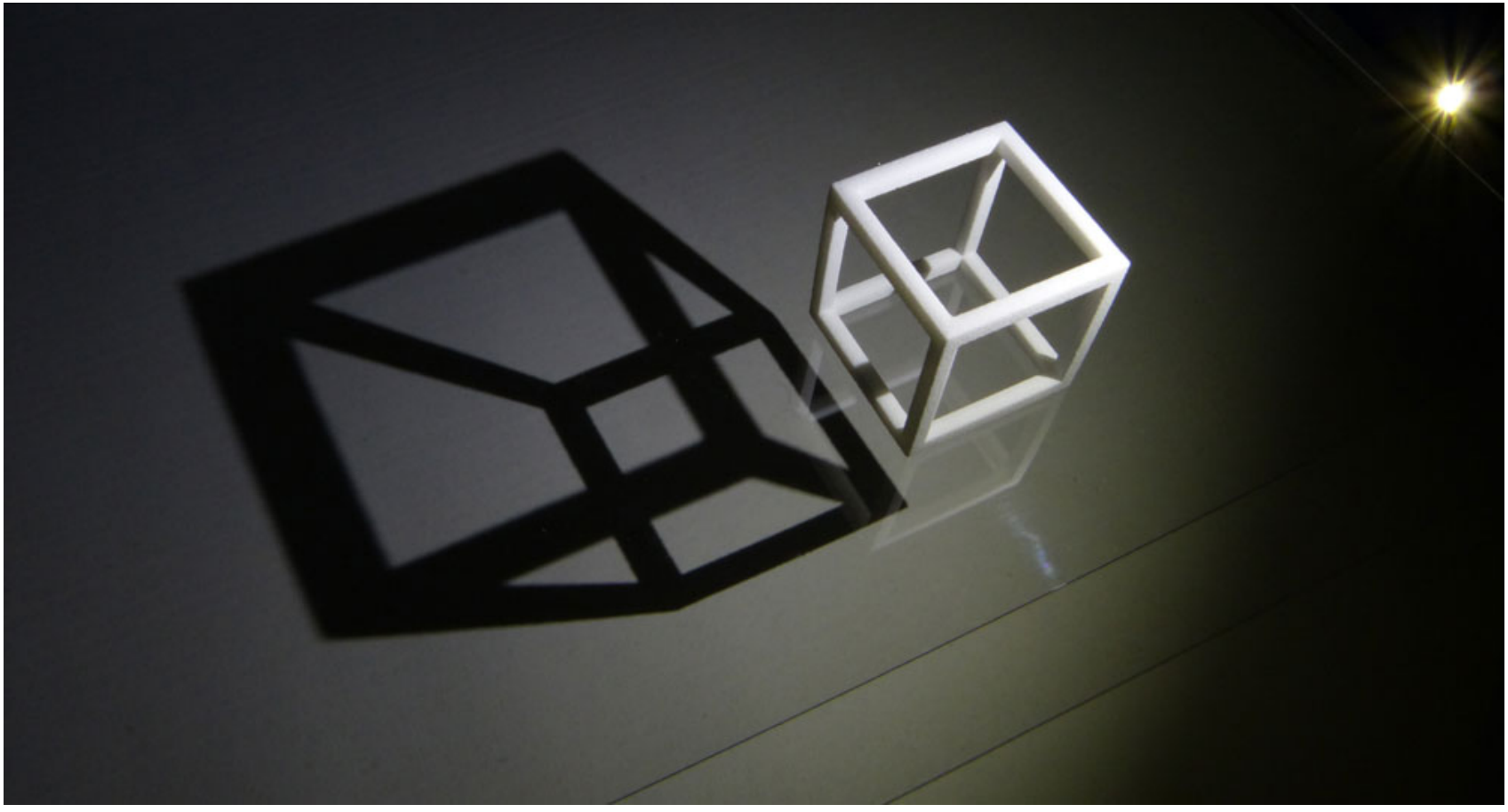


Parallel projection of a hypercube



Hypercube B by Bathsheba Grossman.

Perspective projection of a cube



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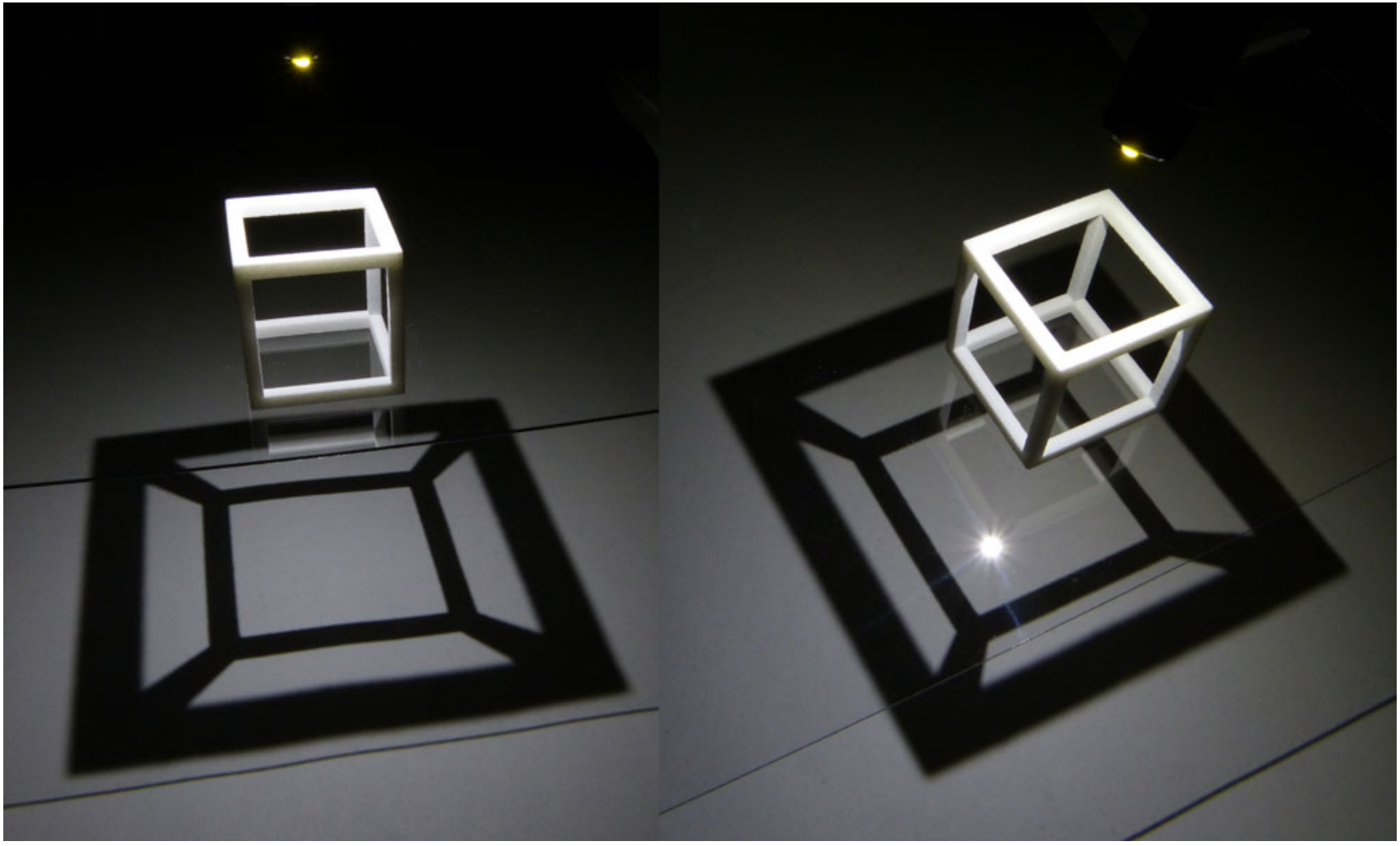

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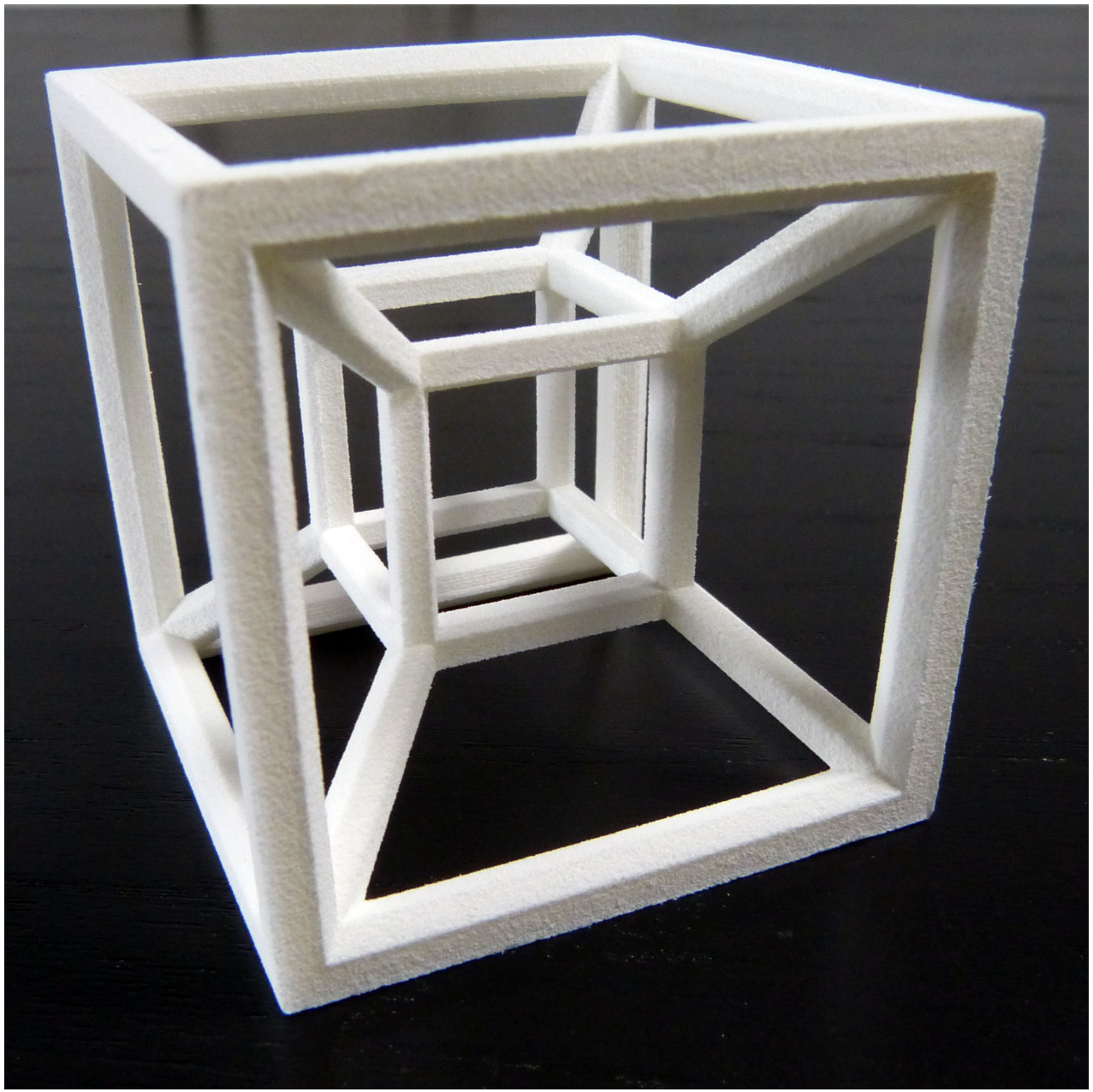
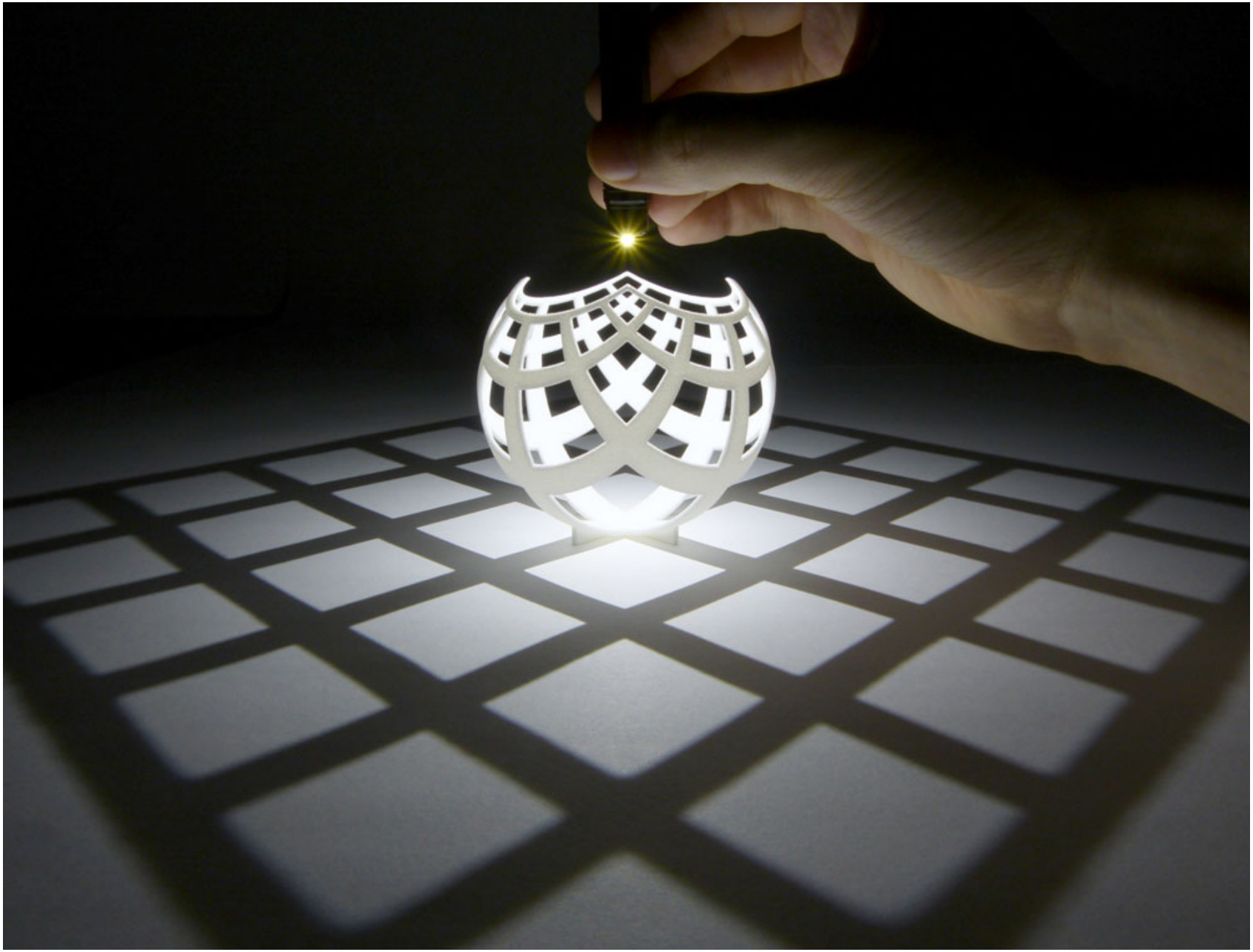
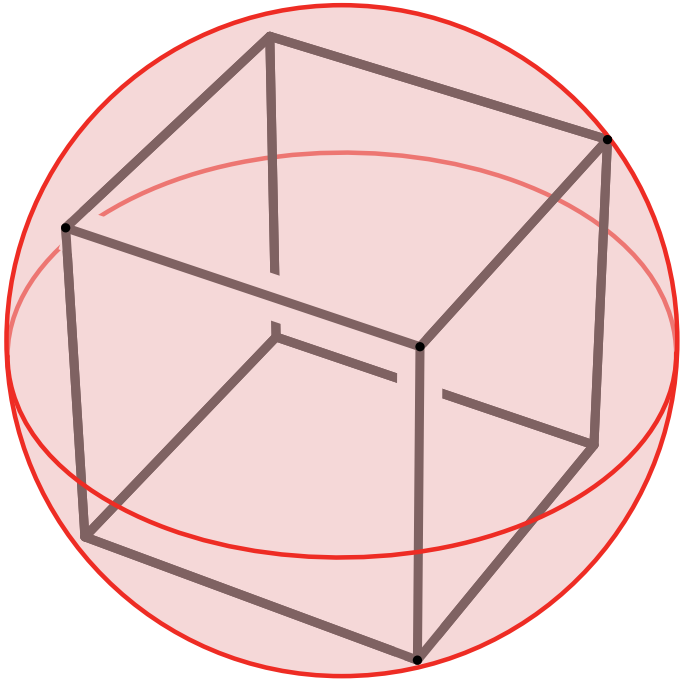


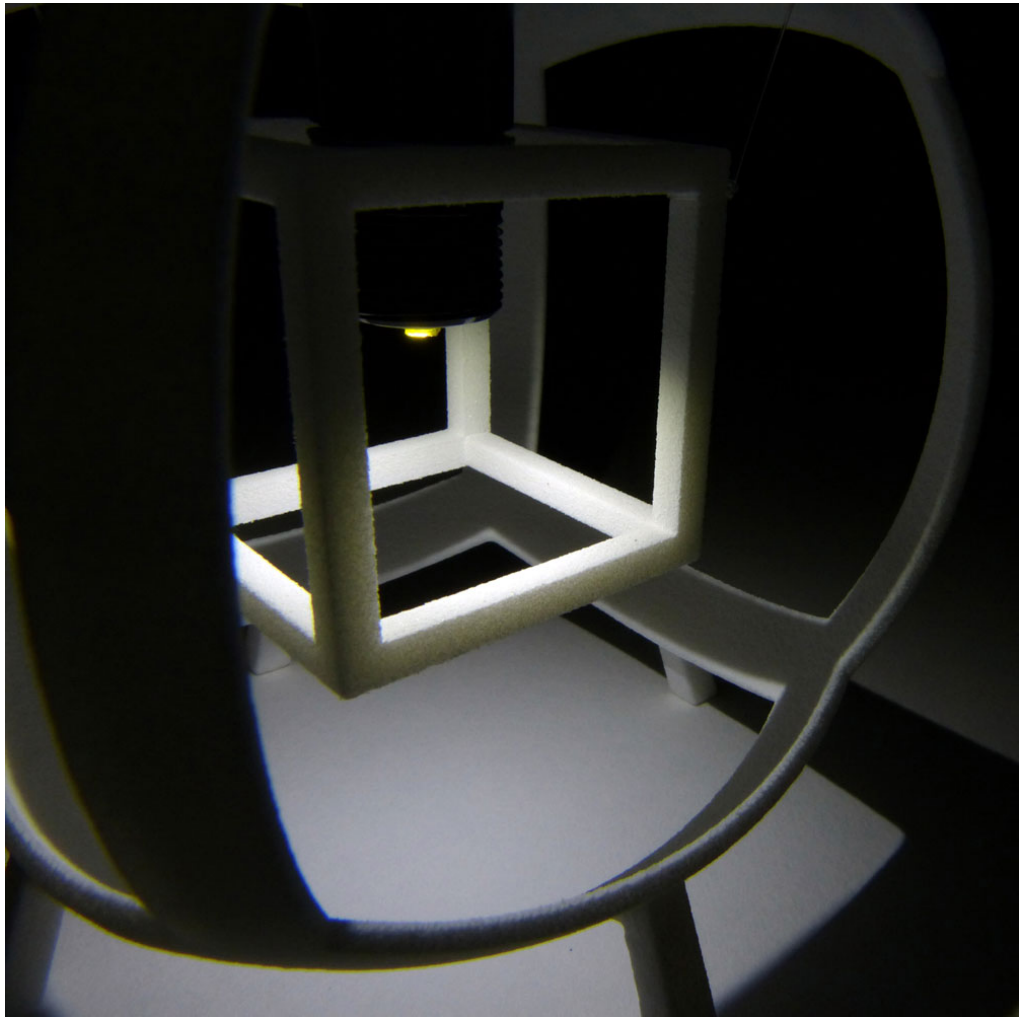
Hypercube A by Bathsheba Grossman.

Stereographic projection



A better method: radially project the cube to the sphere...





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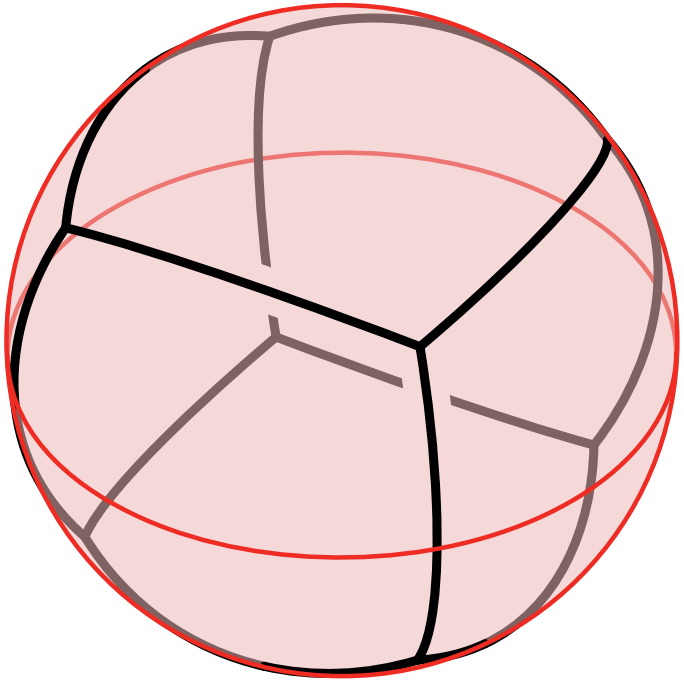
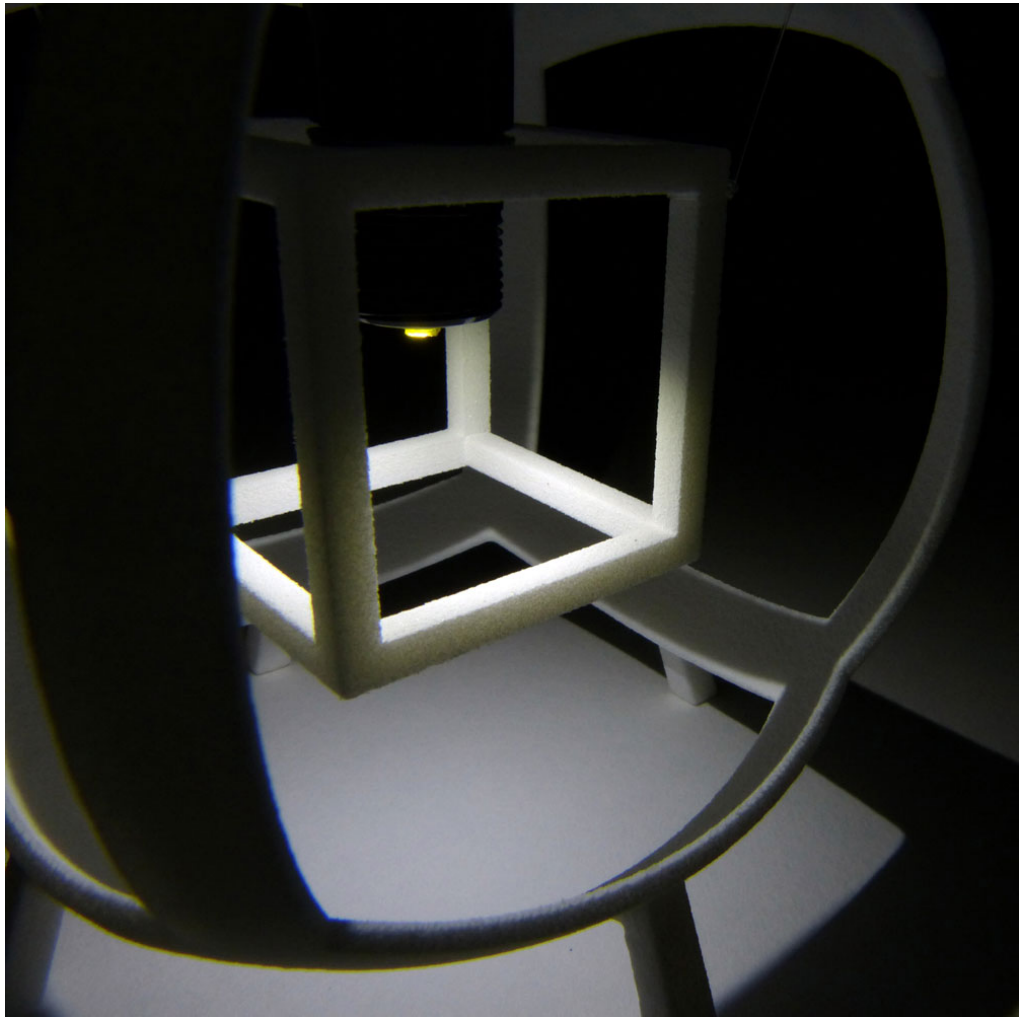
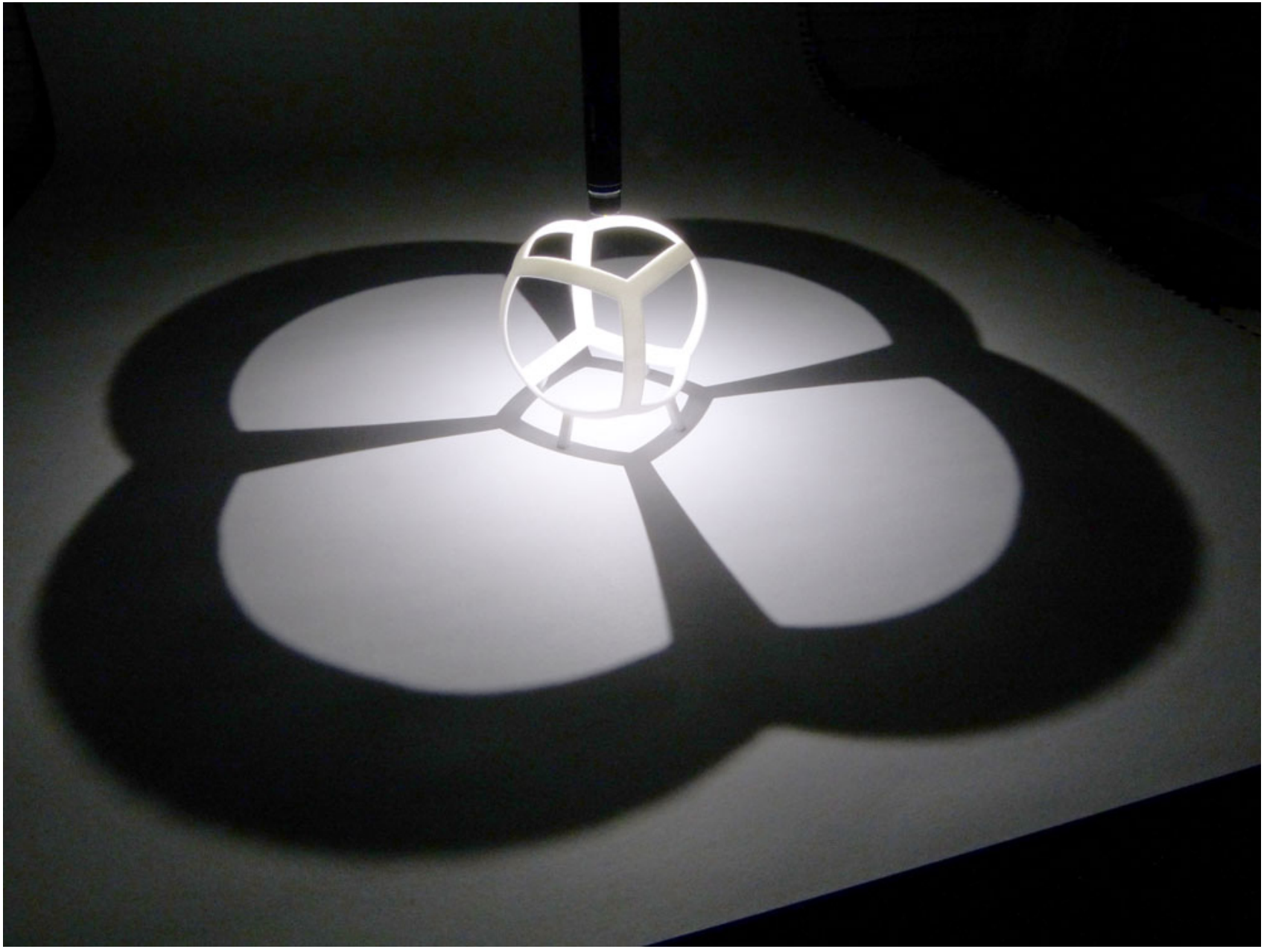




...then stereographically project to the plane



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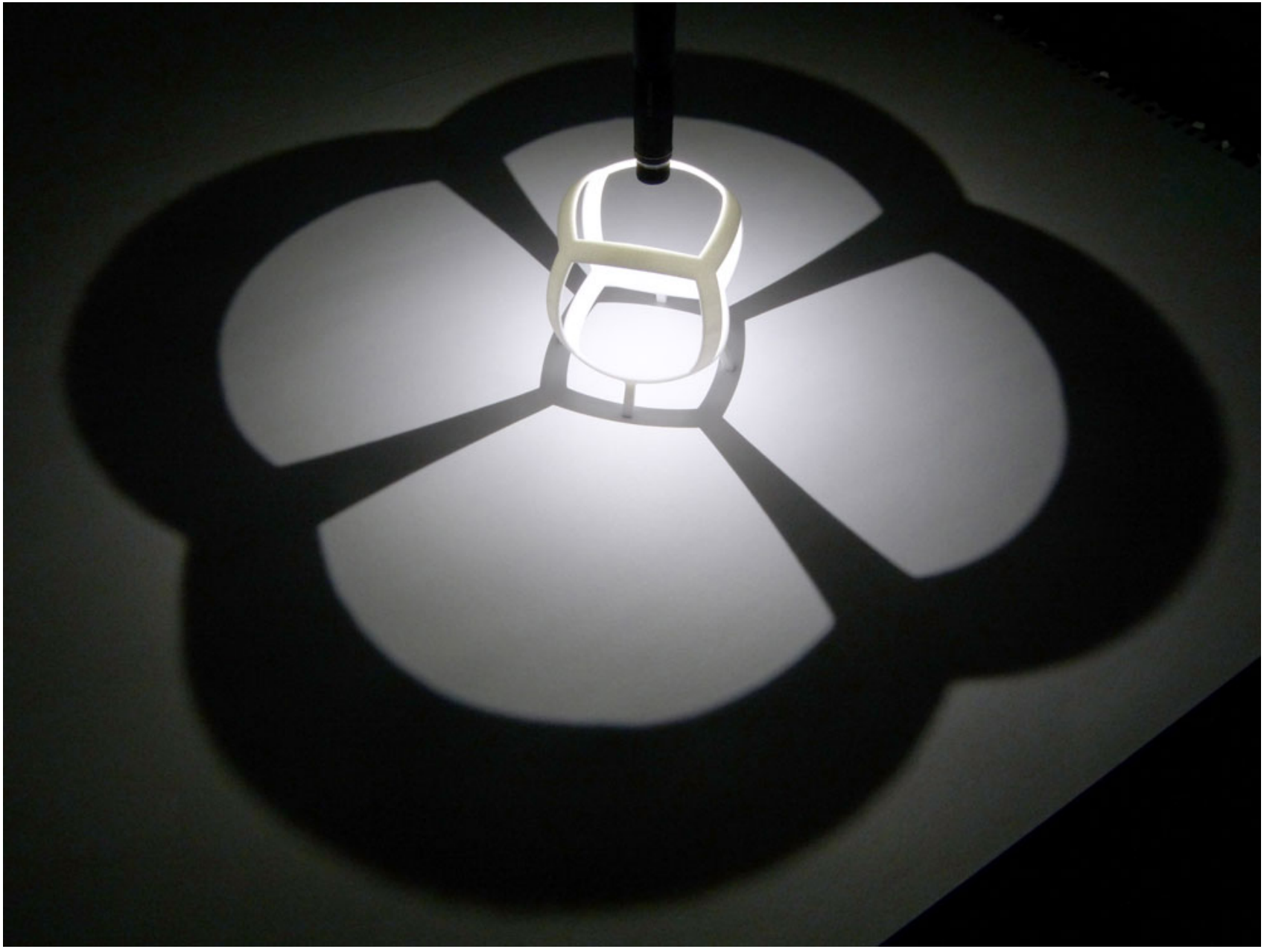
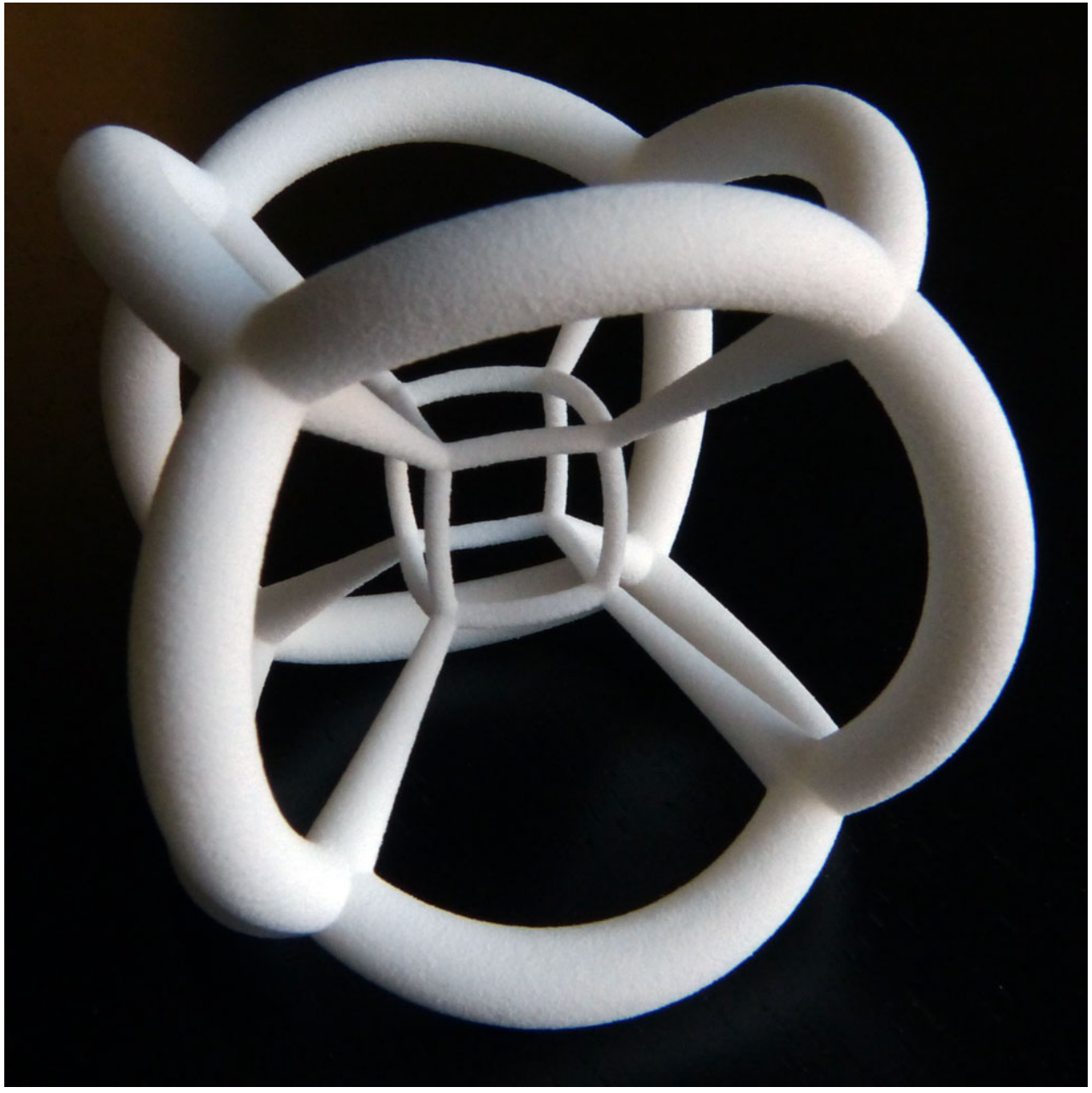


Do the same thing one dimension up for a hypercube



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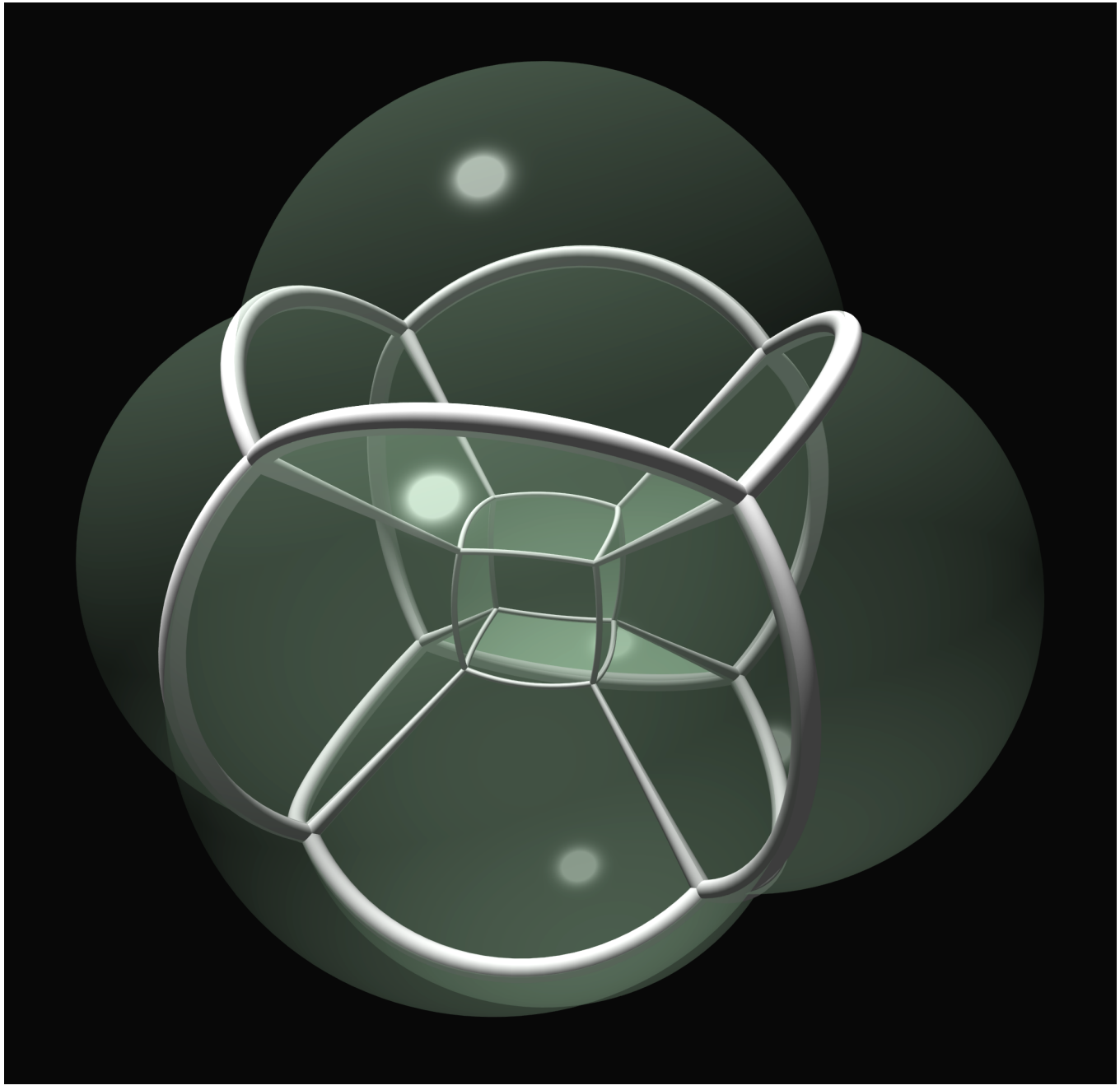
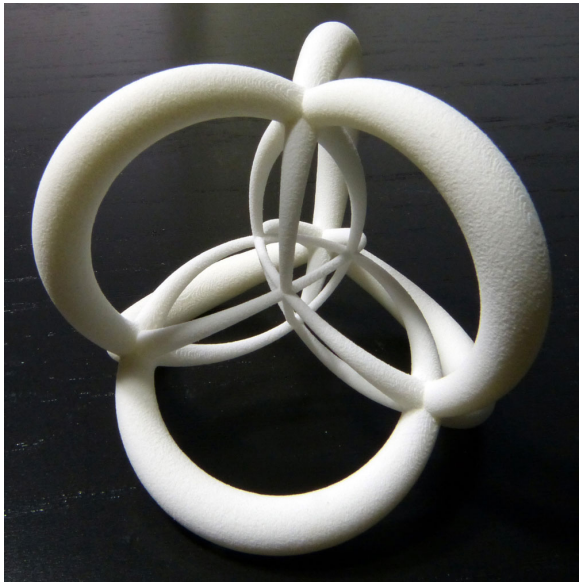
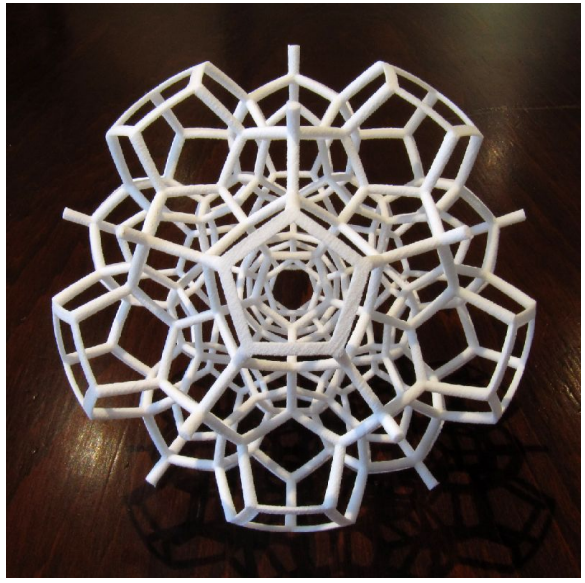


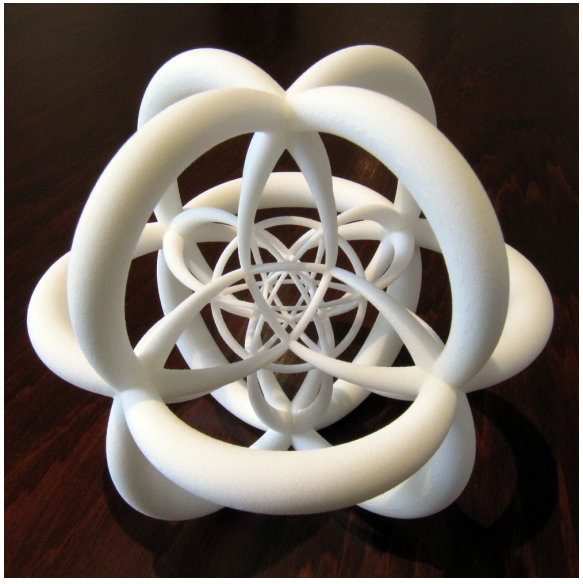
More regular 4-dimensional polytopes



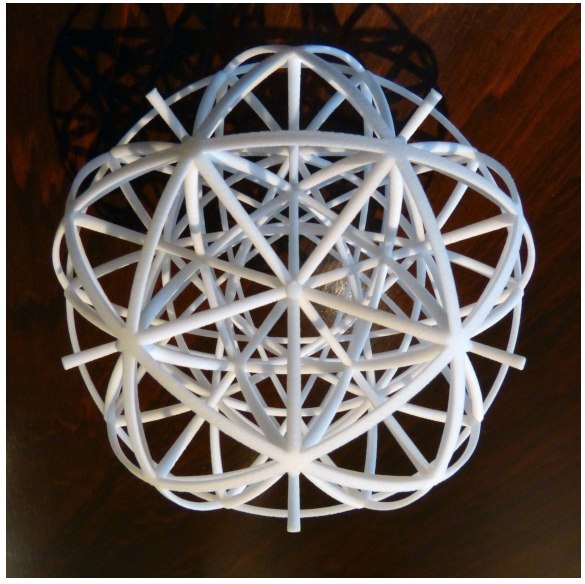
16-cell



Half of a 120-cell



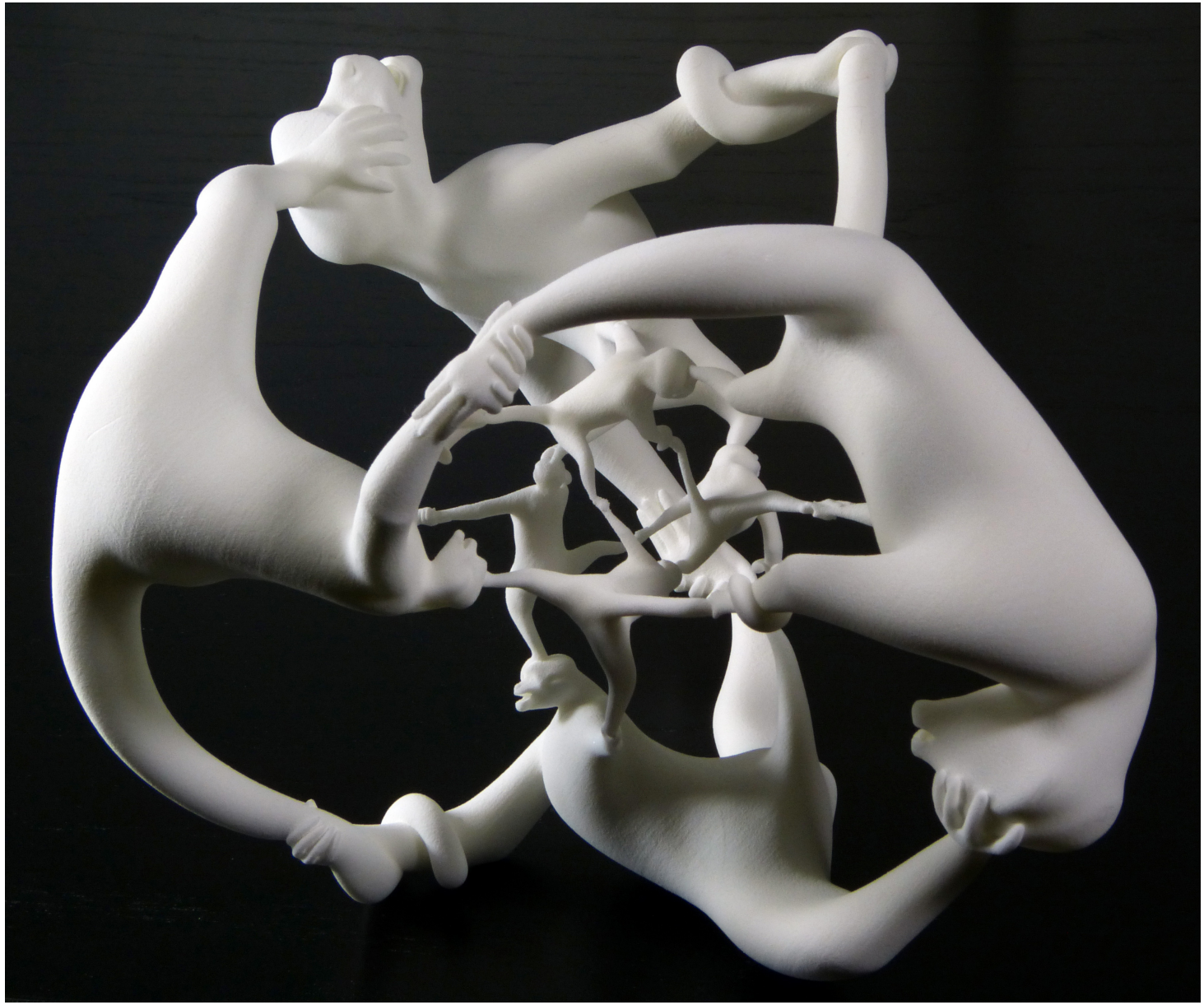
24-cell

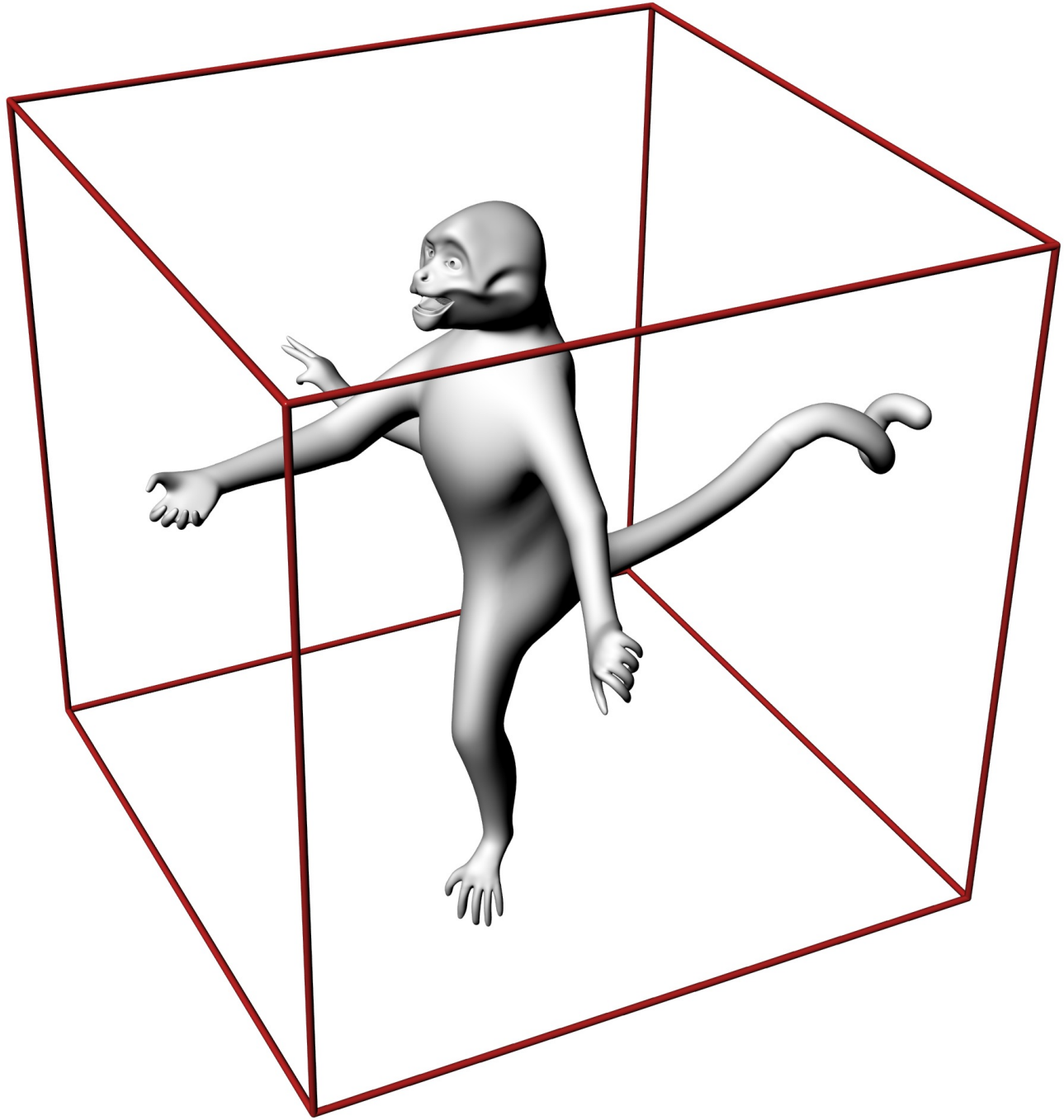


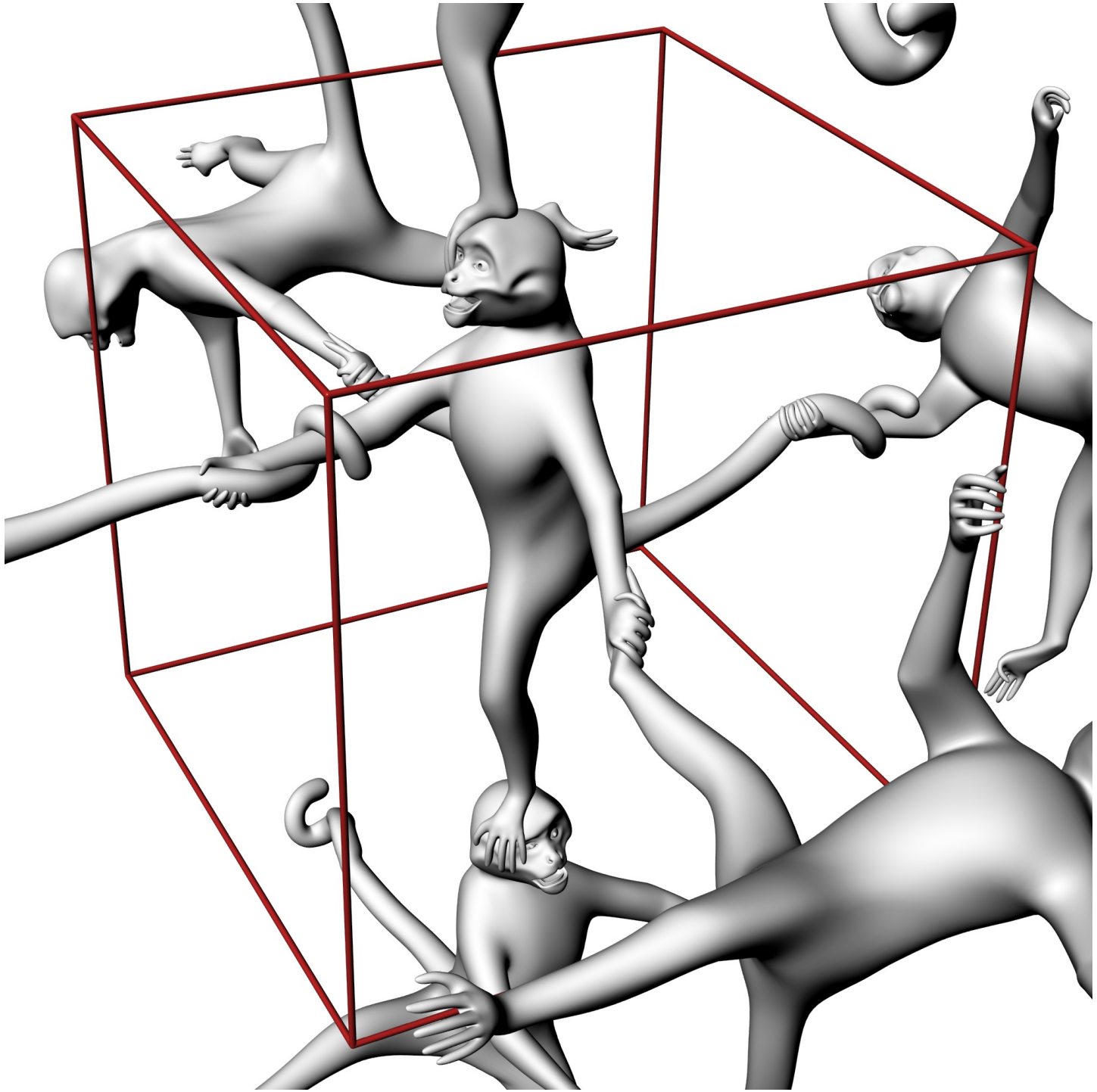
Half of a 600-cell

Quintessence (joint work with Saul Schleimer)

More fun than a hypercube of monkeys (joint work with Will Segerman)





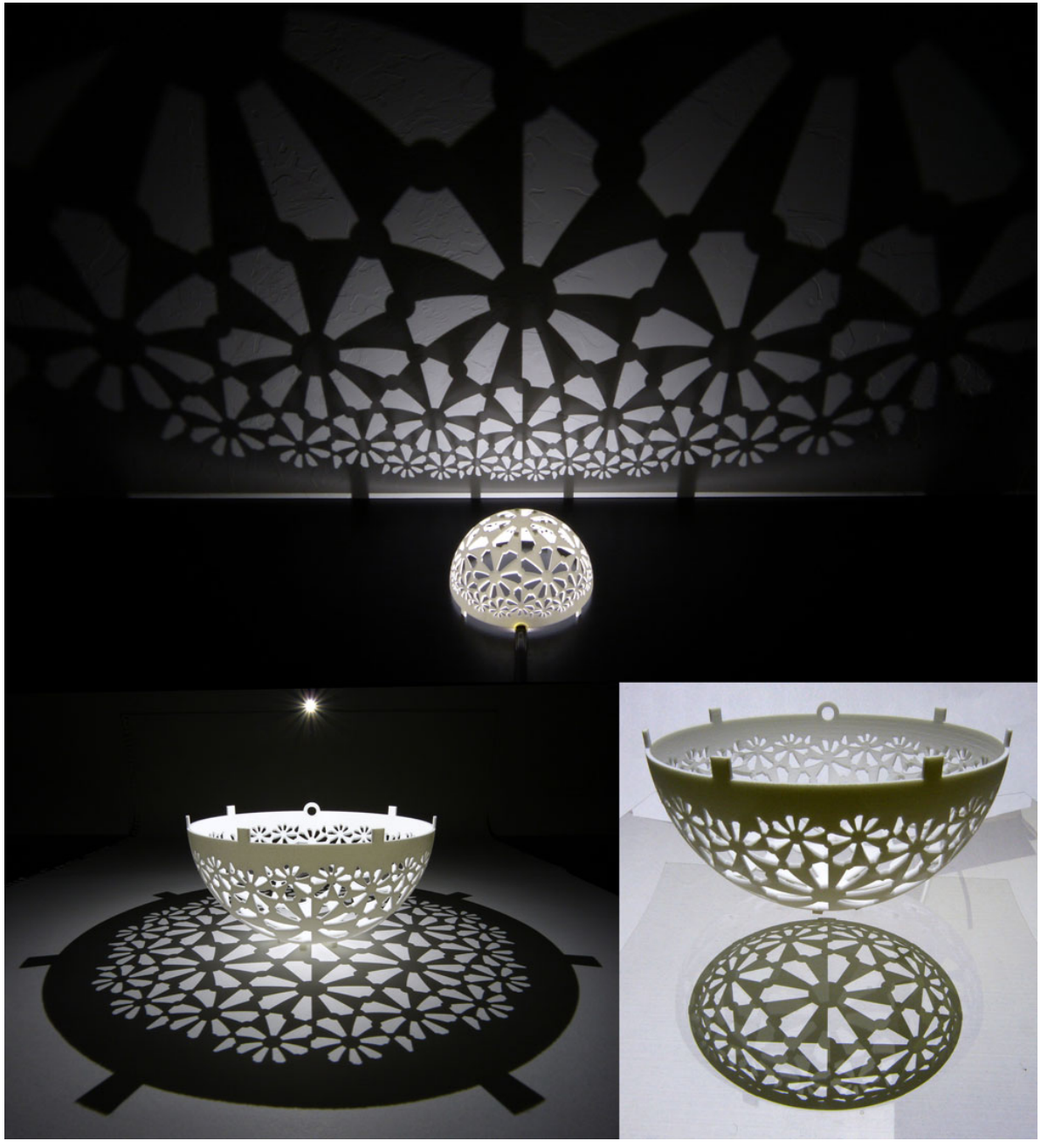


http://monkeys.hypernom.com

http://monkeys.hypernom.com

http://monkeys.hypernom.com

Models of hyperbolic geometry



Mobiles (joint work with Marco Mahler)

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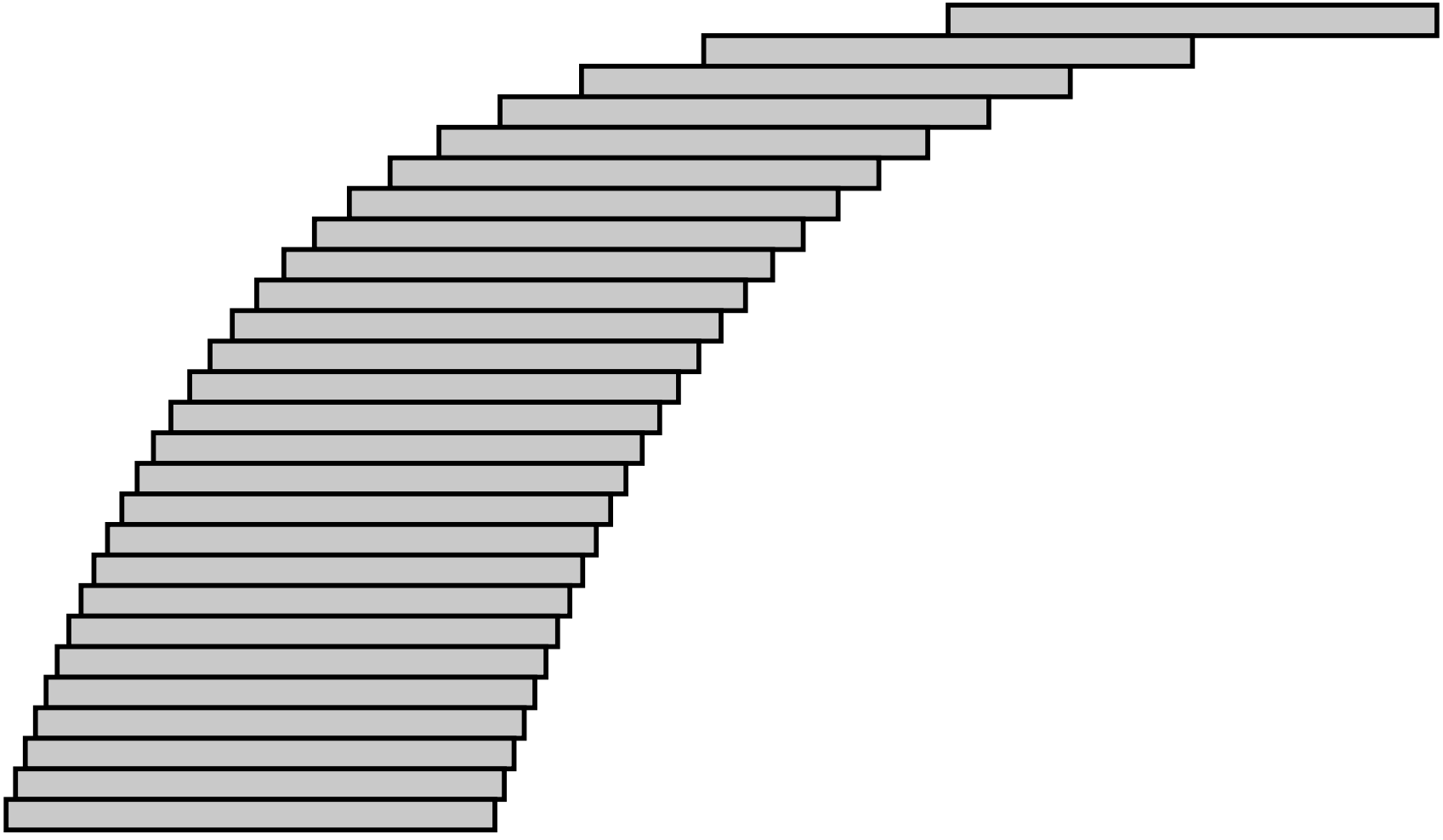

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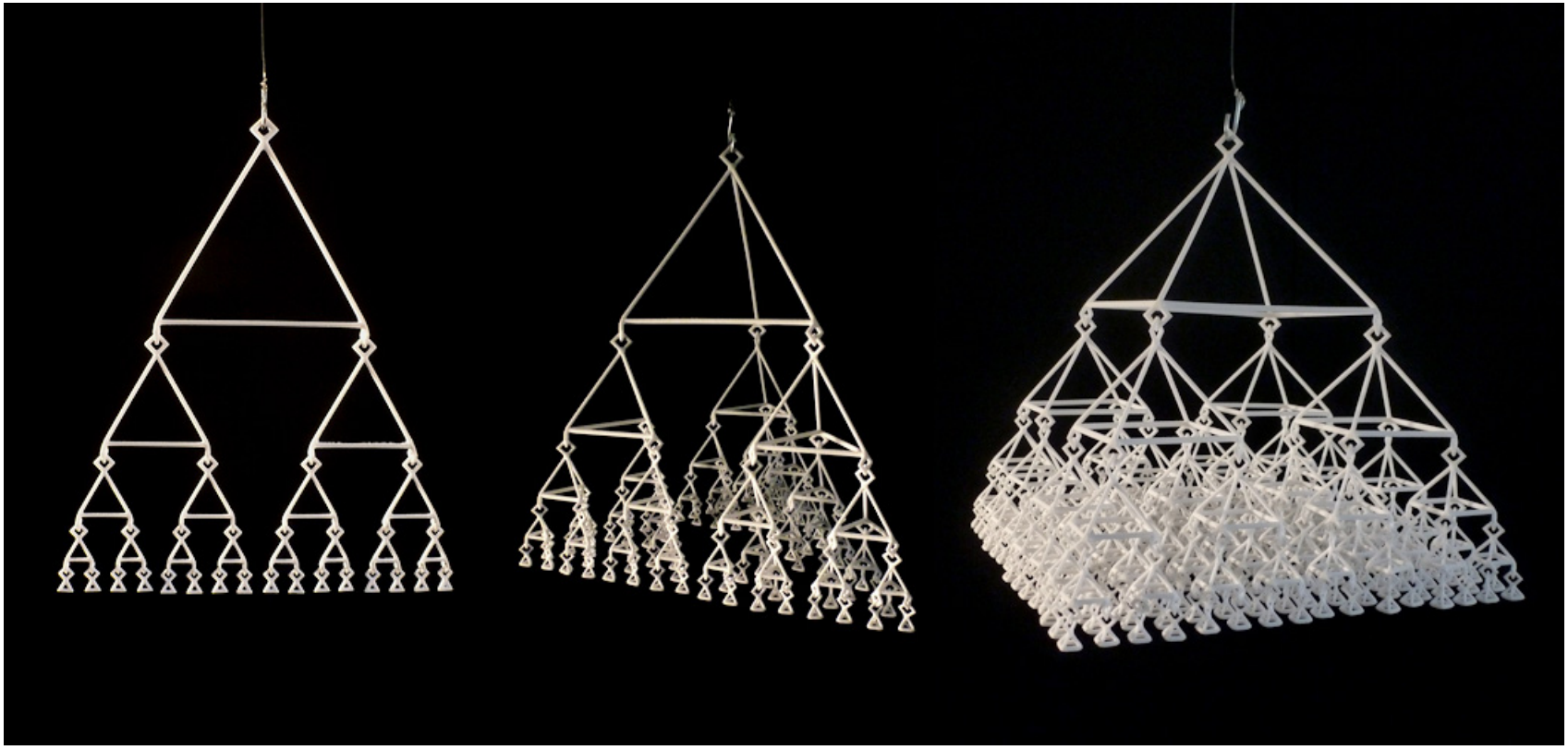

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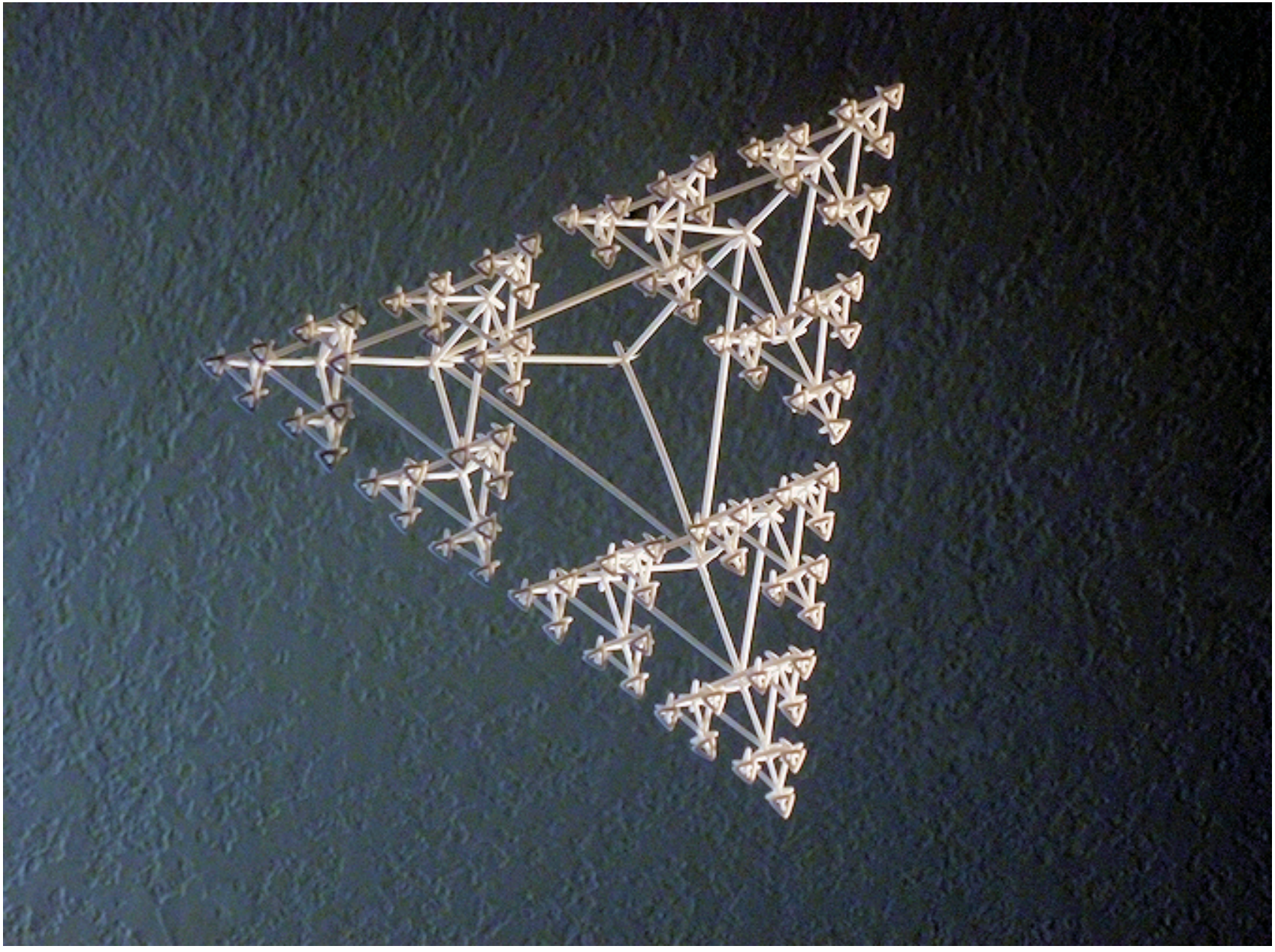
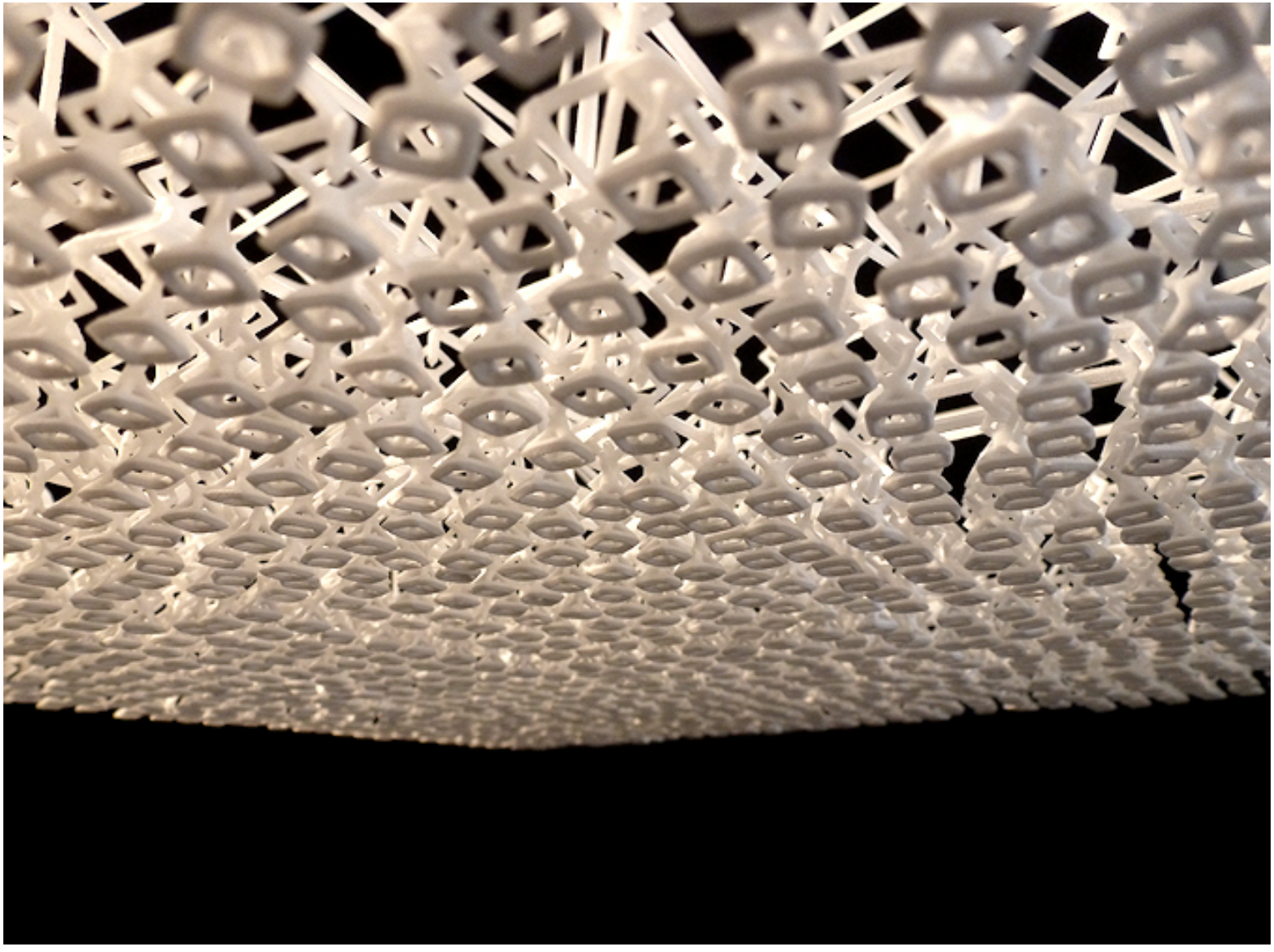
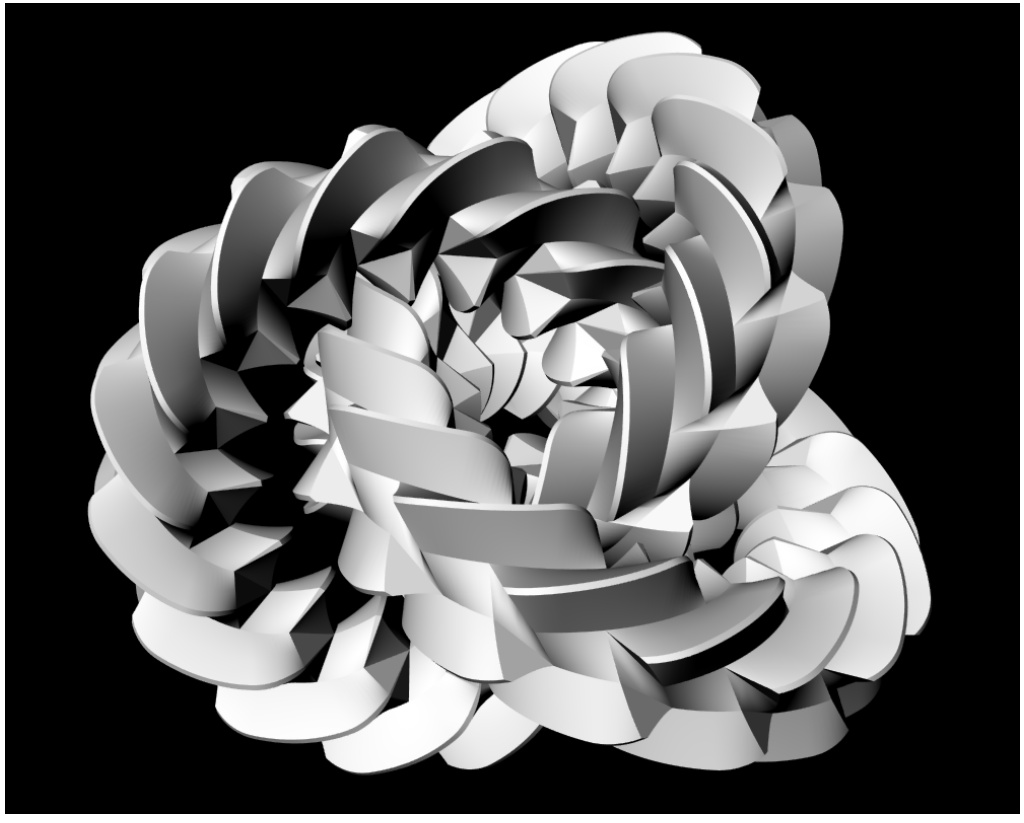


Mobiles (joint work with Marco Mahler)



Triple gear (joint work with Saul Schleimer)



Manchester Metroshuttle advertisement, Photo credit: Bill Beaty

Photo credit: meladramos of reddit.

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Photo credit: meladramos of reddit.

Three pairwise meshing gears are usually frozen...

Manchester Metroshuttle advertisement, Photo credit: Bill Beaty

Photo credit: meladramos of reddit.

Three pairwise meshing gears are usually frozen...

A challenge: Find a triple of pairwise meshing gears that moves! Is there a way to make them work if the gears are non-planar?

"Umbilic Rolling Link" by Helaman Ferguson.

"Knotted Gear" by Oskar van Deventer.

Our solution is inspired by these "linked" gears.

"Umbilic Rolling Link" by Helaman Ferguson.

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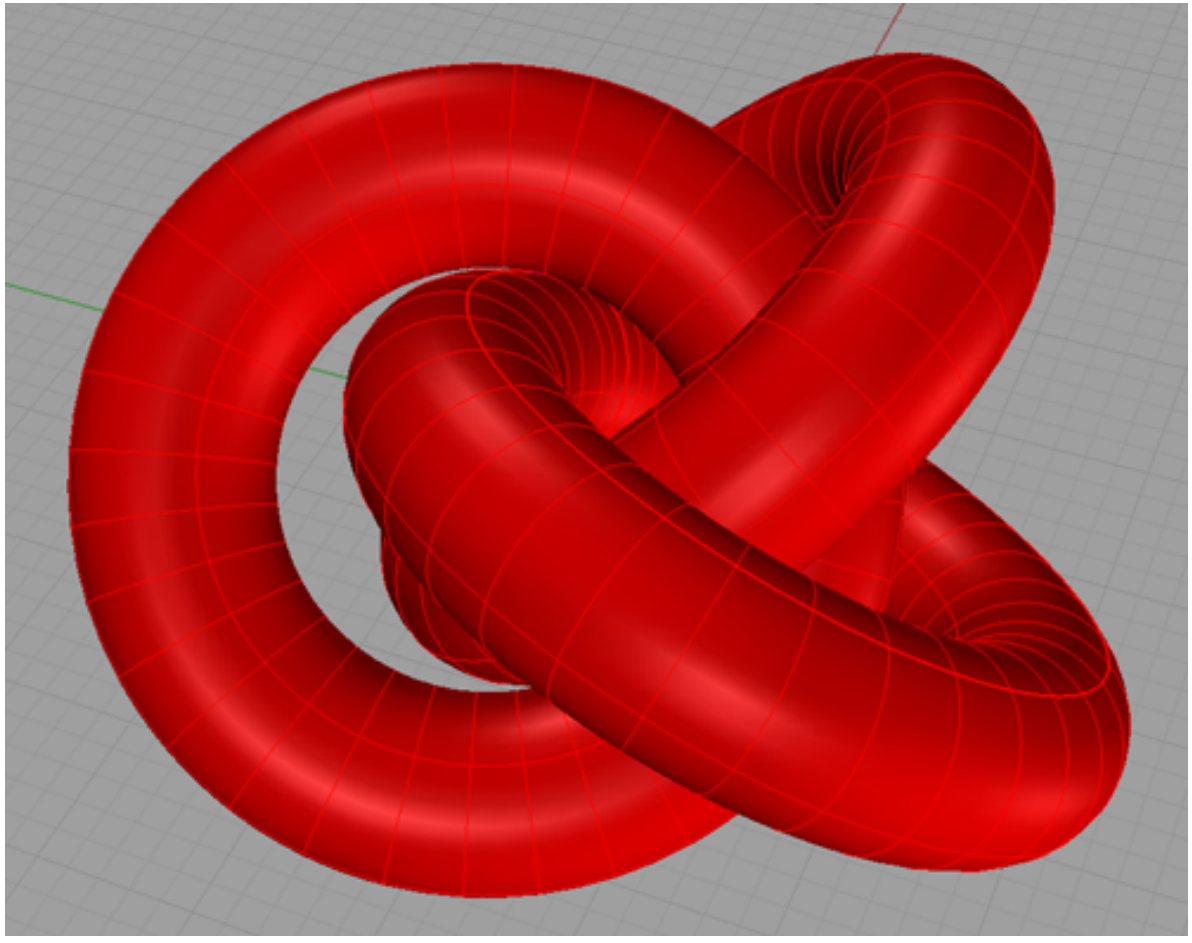
Our solution is inspired by these "linked" gears.

They have two "gears"; we want to do the same with three.

We chose the three-component Hopf link as the basis of the design.

We gradually inflate the three rings, letting them bump against each other while preserving the 3-fold symmetry, until they reach maximum thickness.



We had hoped that these rings would only be able to rotate along their axes.

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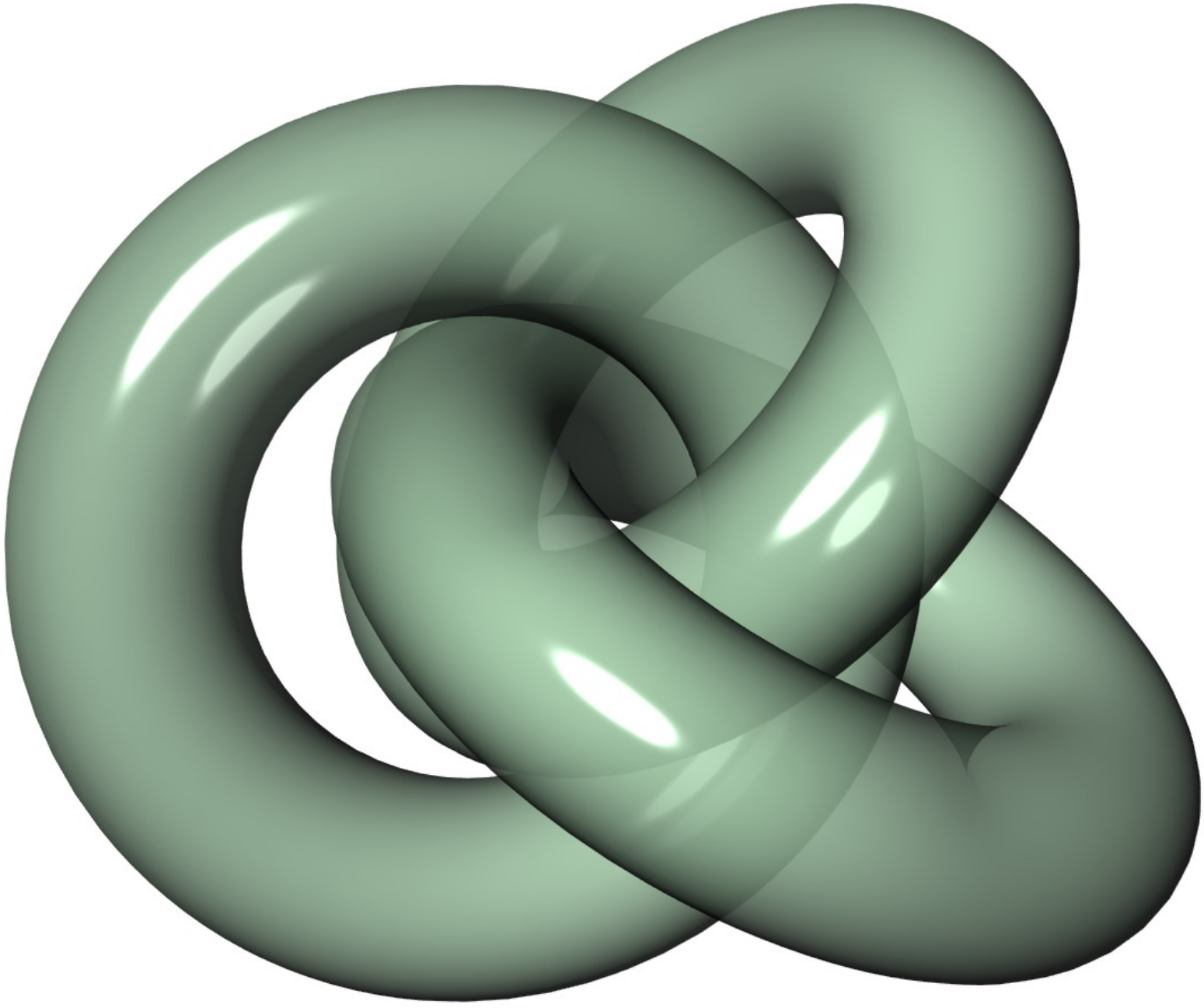
To stop them moving out of place, we design gear teeth.

To design the teeth, we investigate how the rings touch each other.



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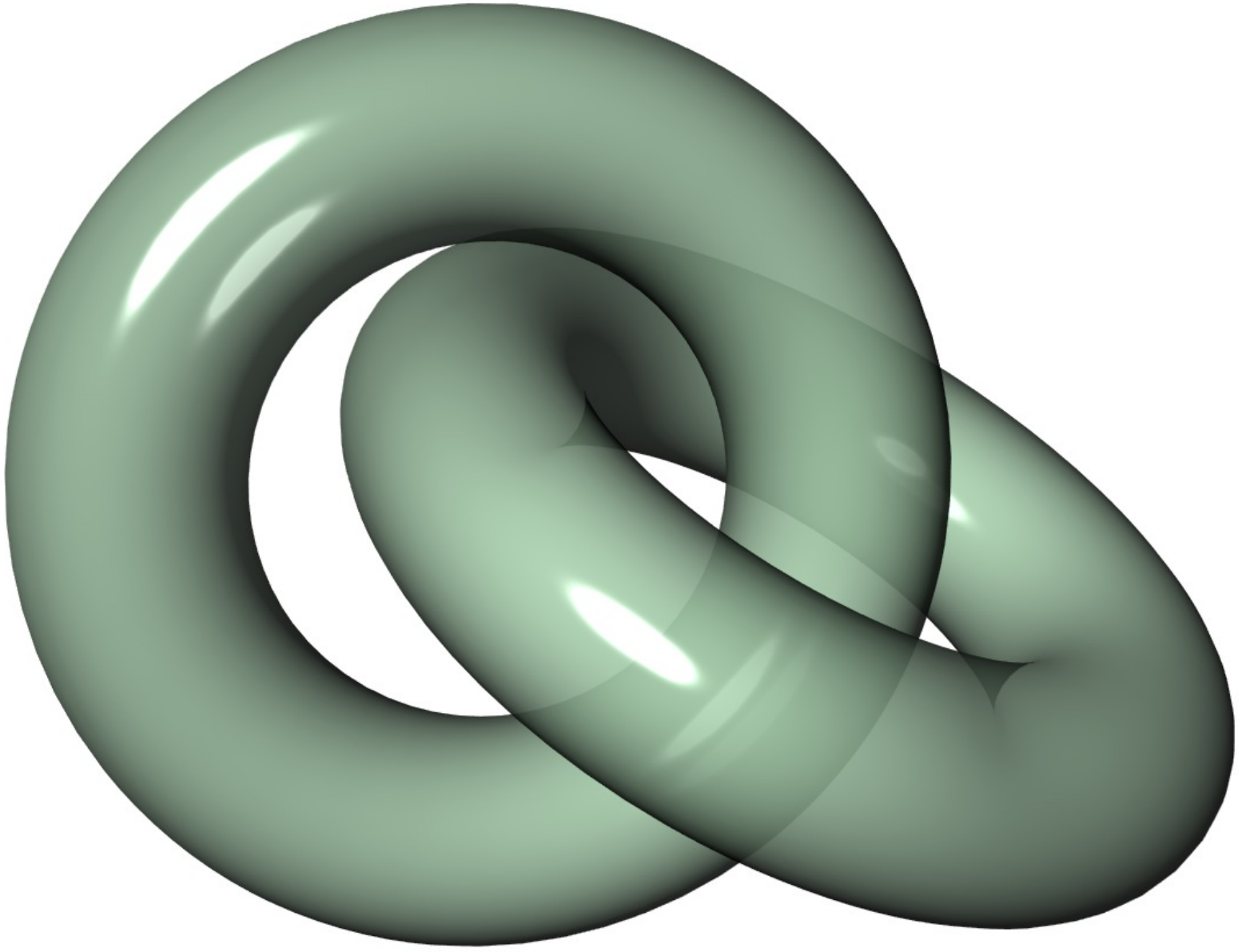

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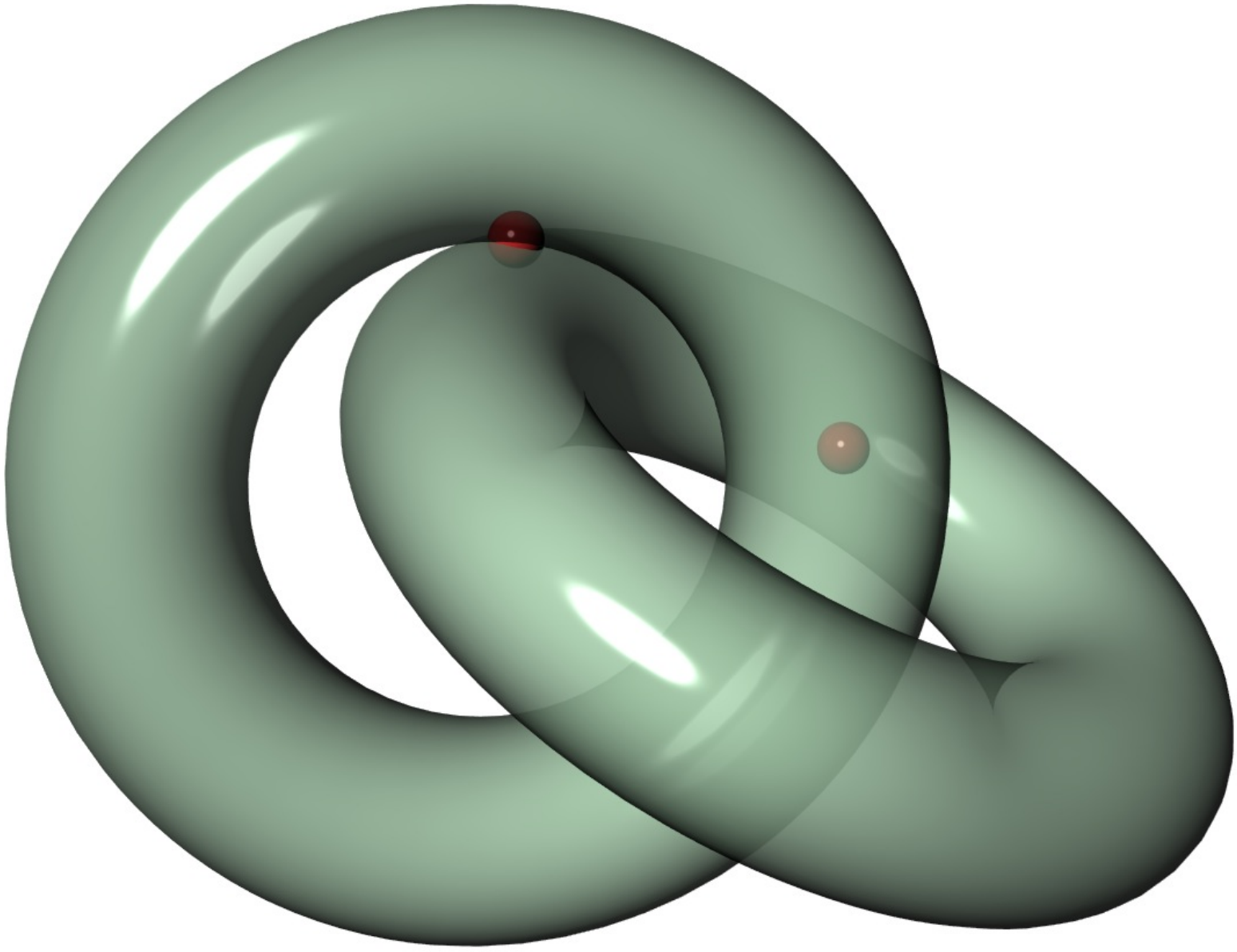

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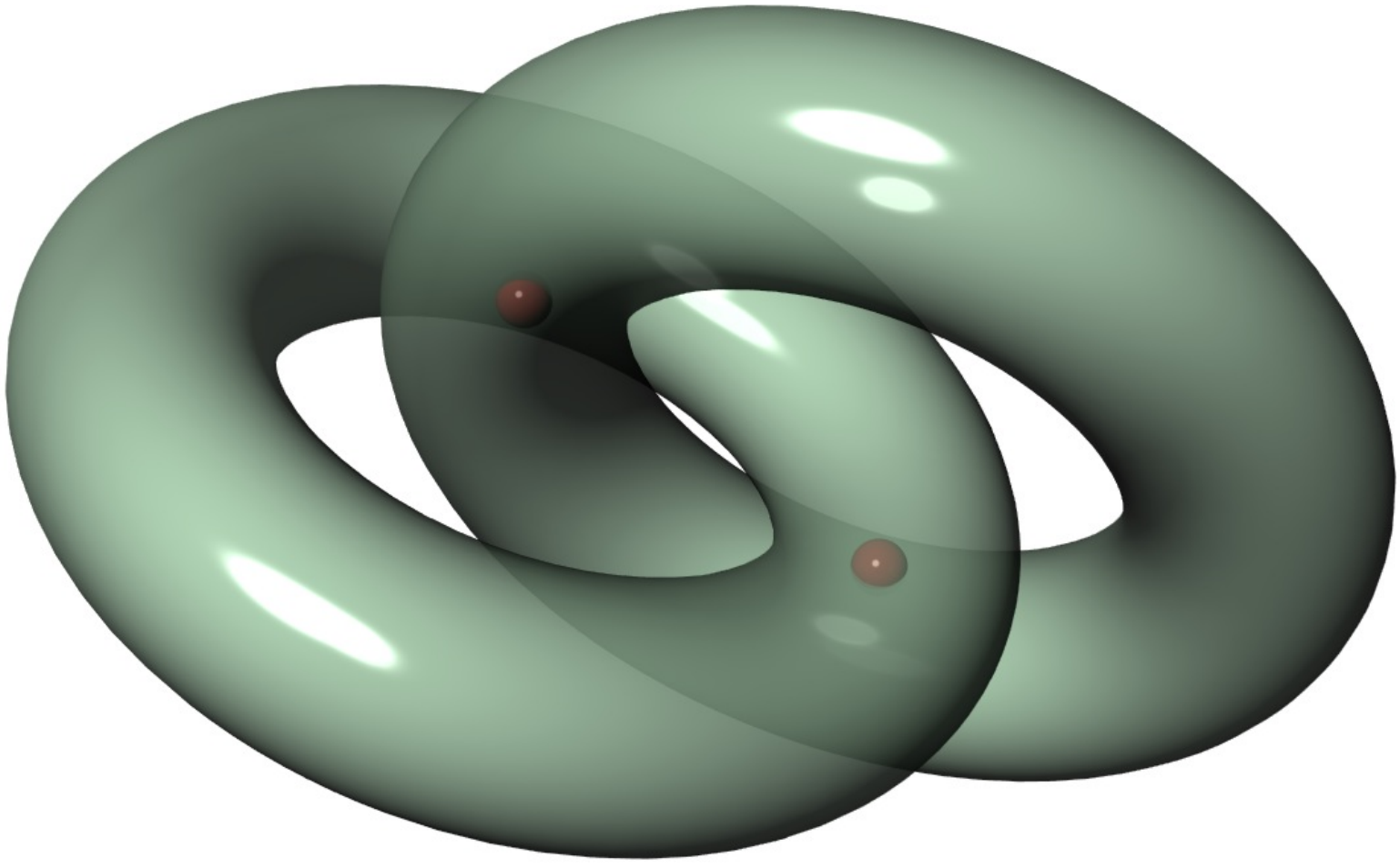

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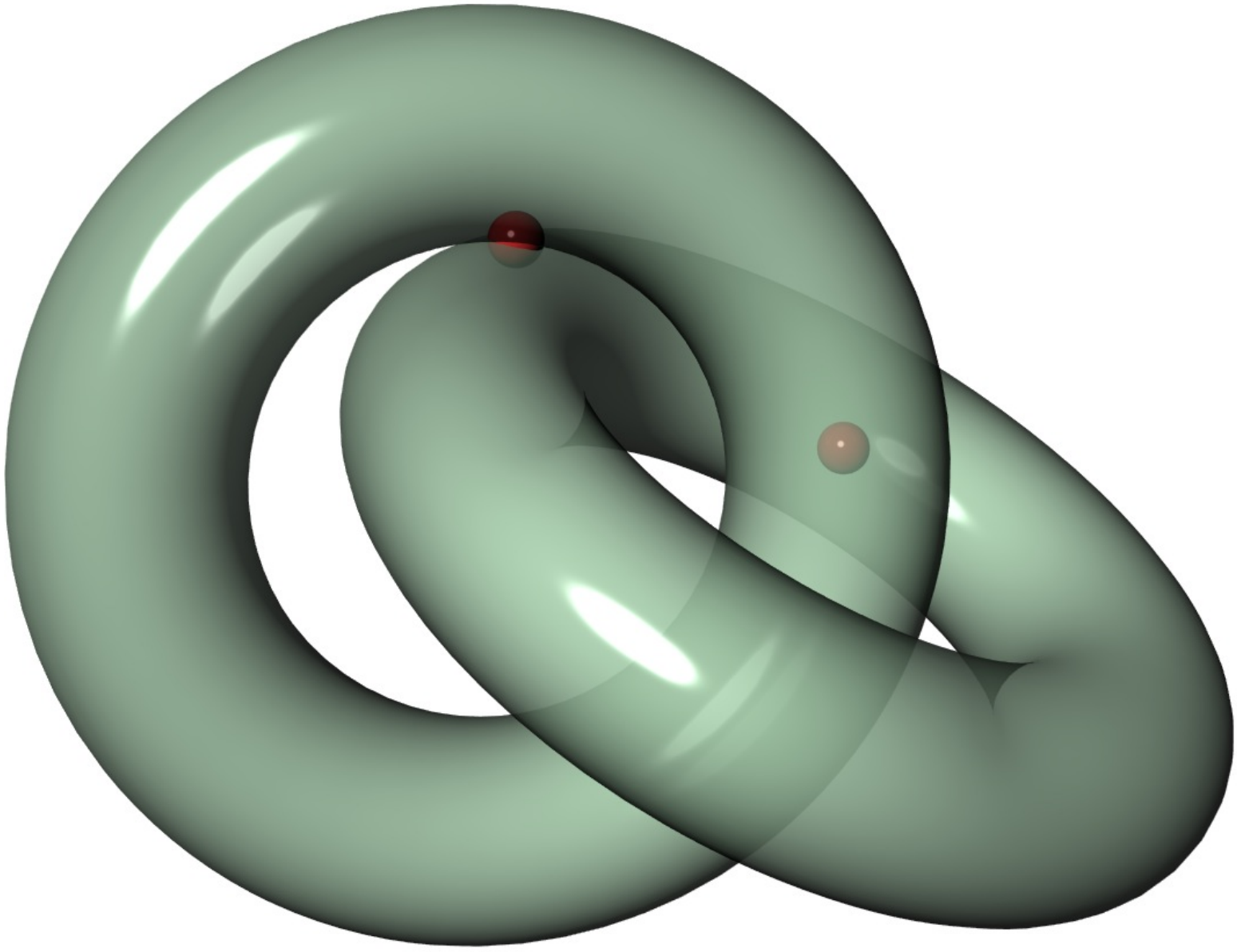

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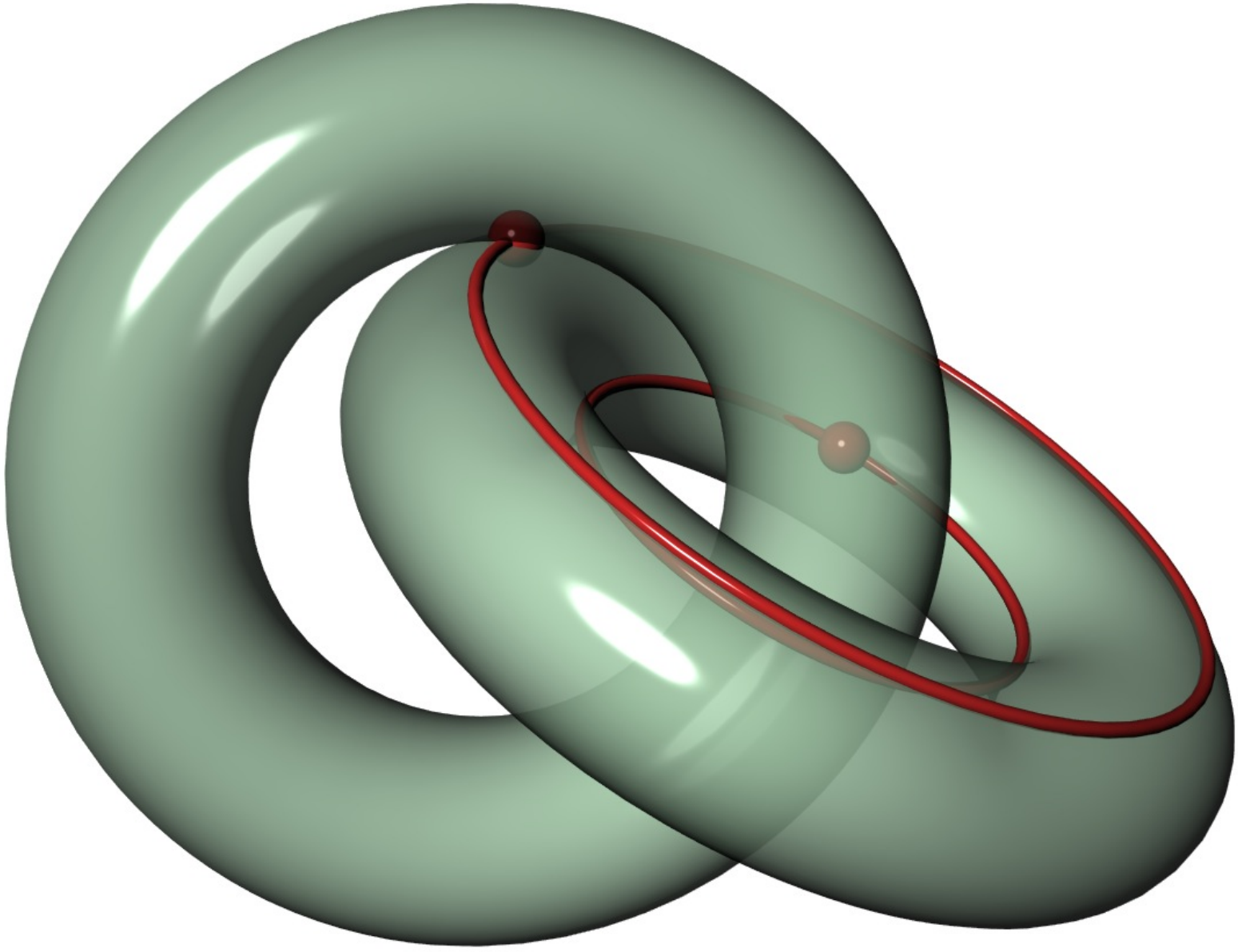

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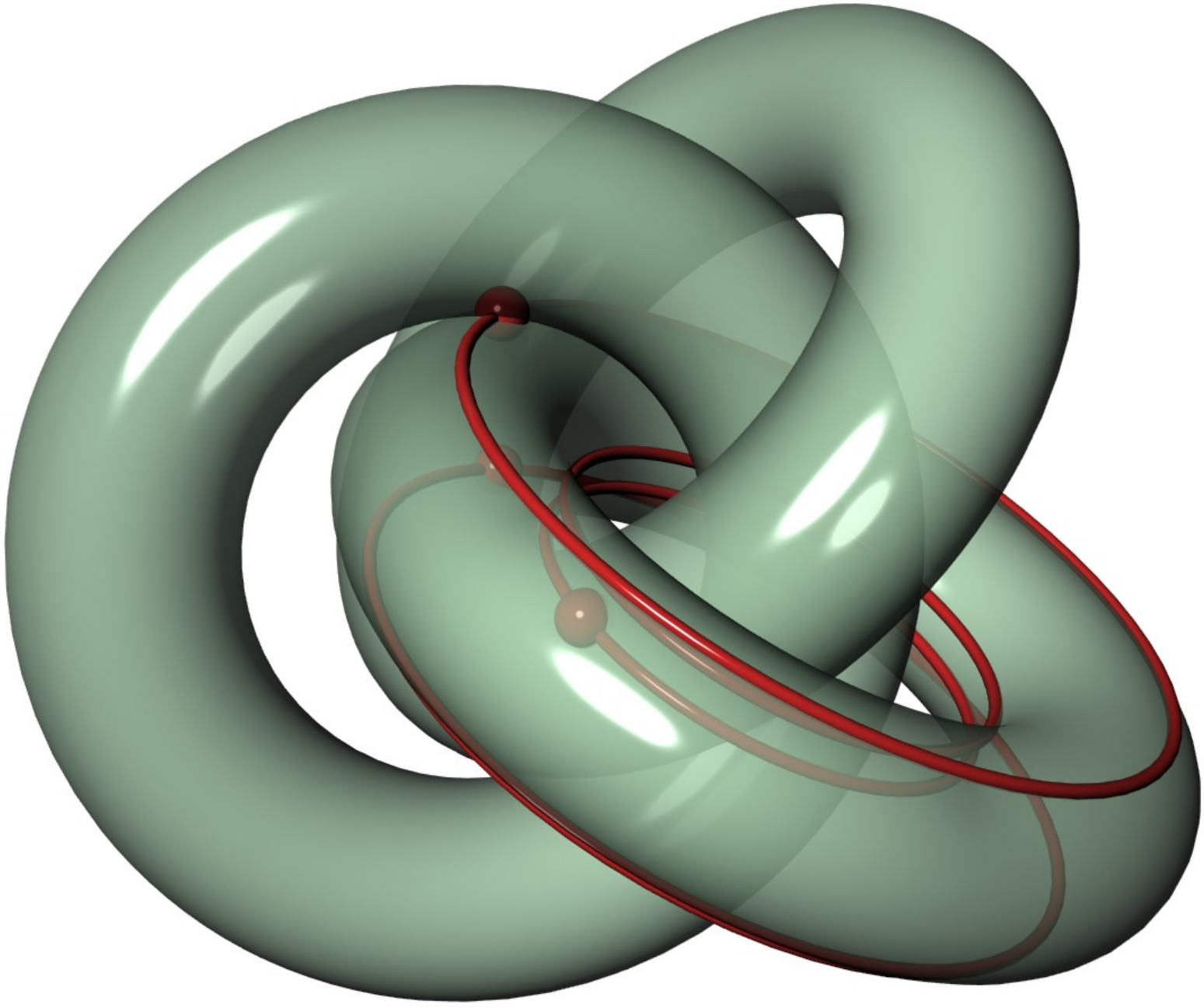

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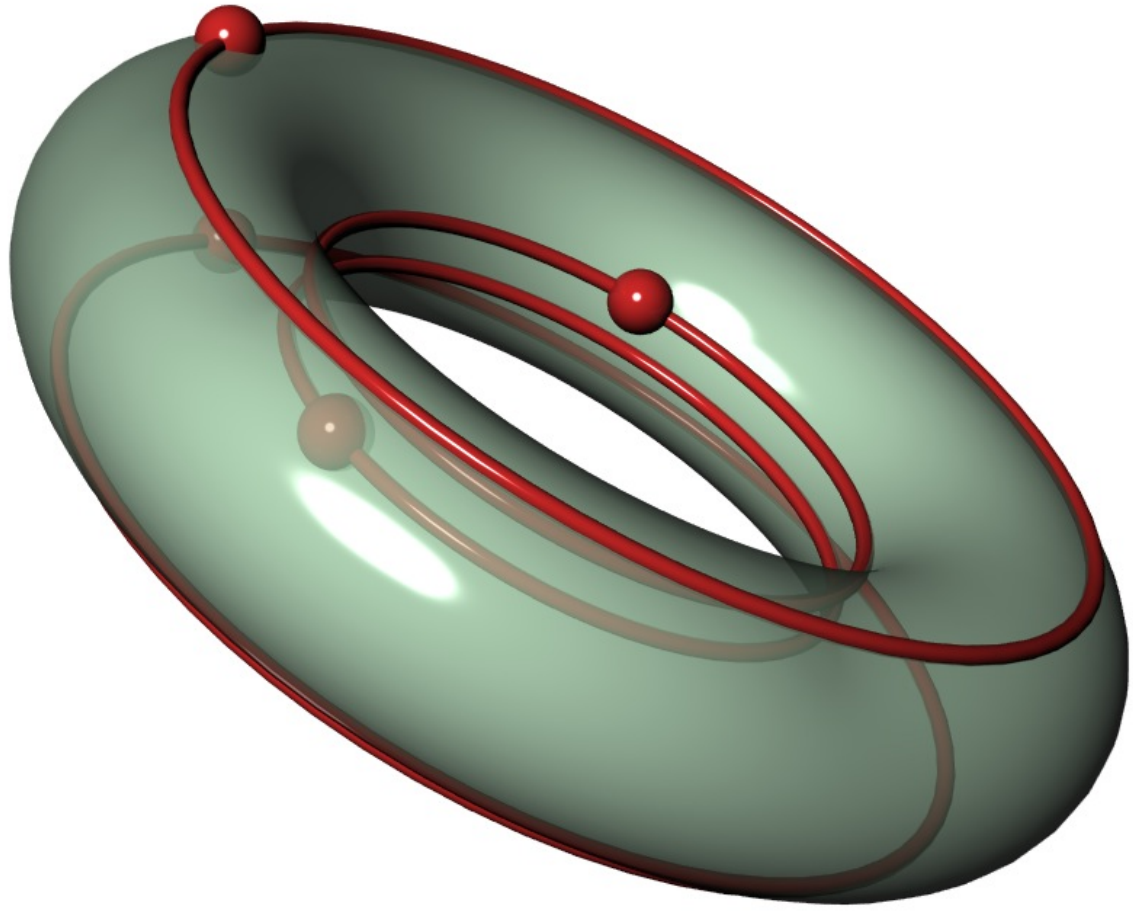

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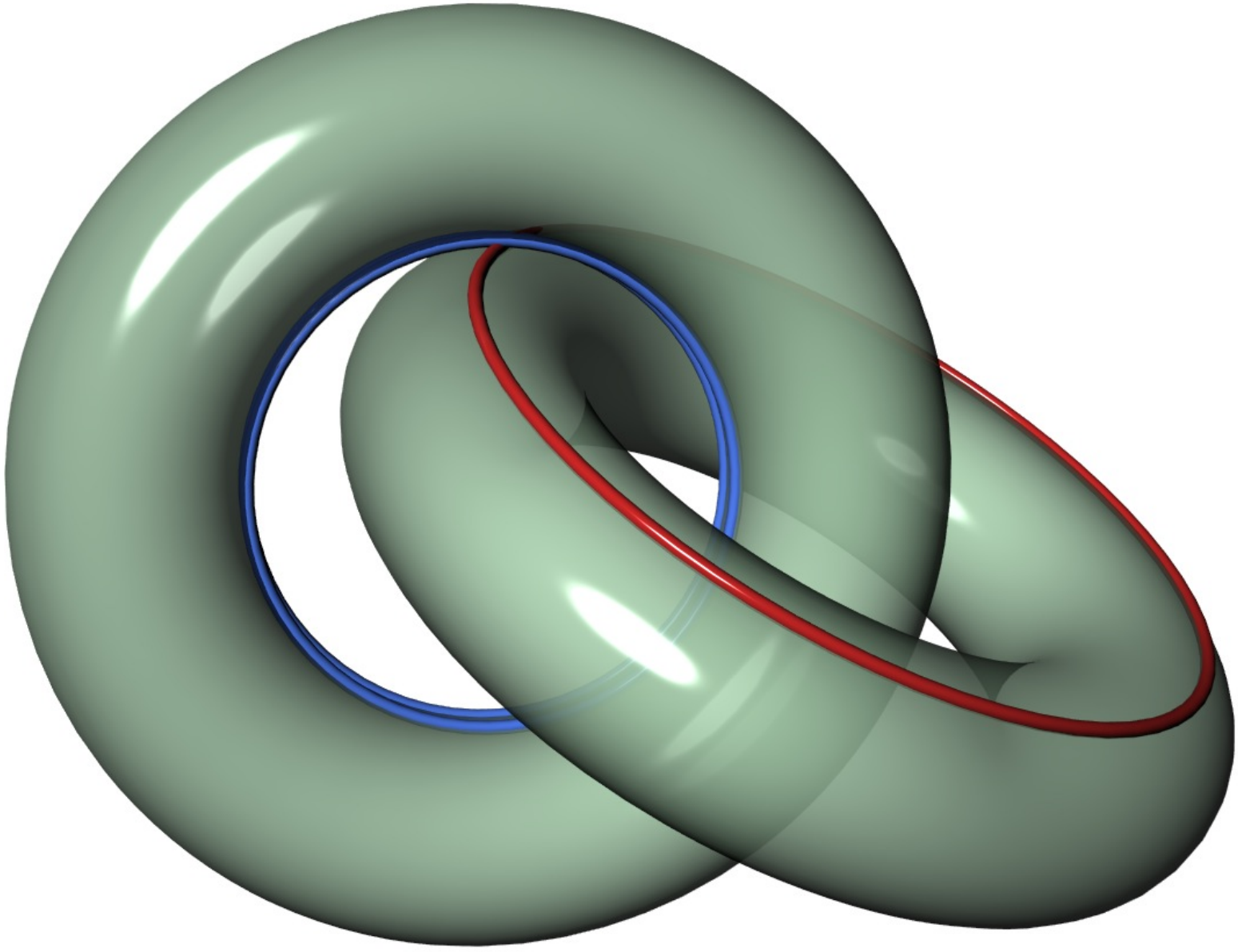
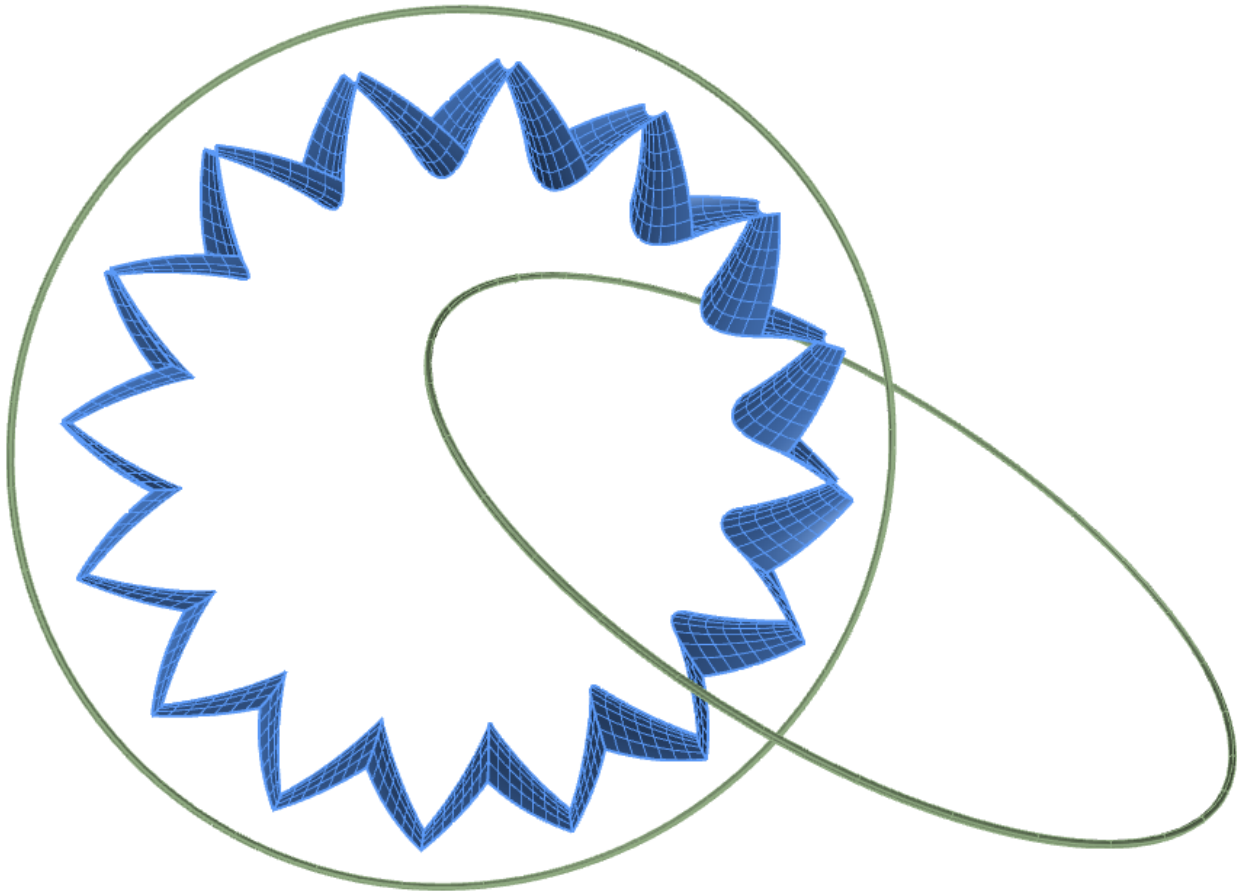
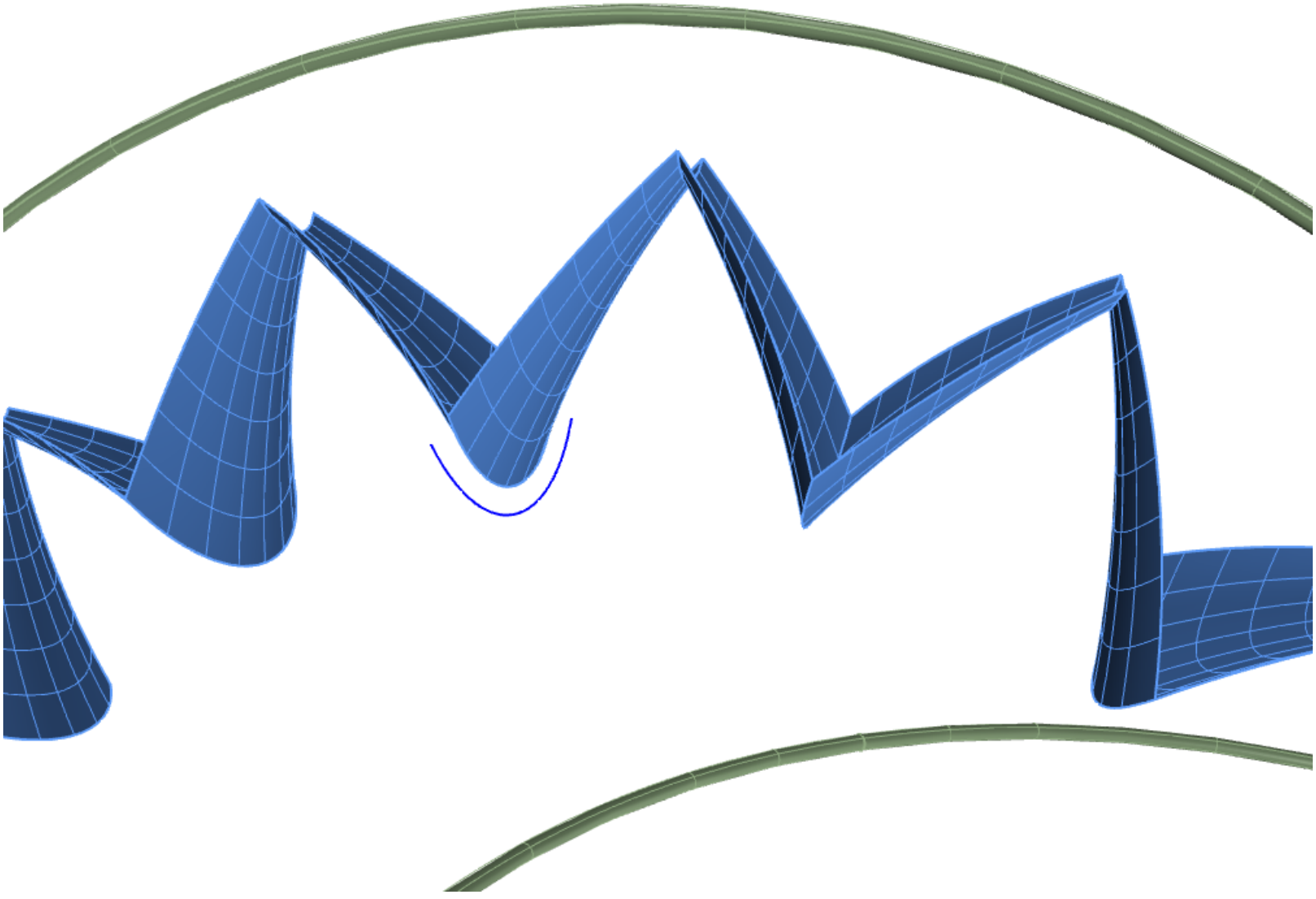


The “inner” teeth are the images of planes in toroidal coordinates.



The “inner” teeth are the images of planes in toroidal coordinates.
The “outer” teeth are determined by “carving”.



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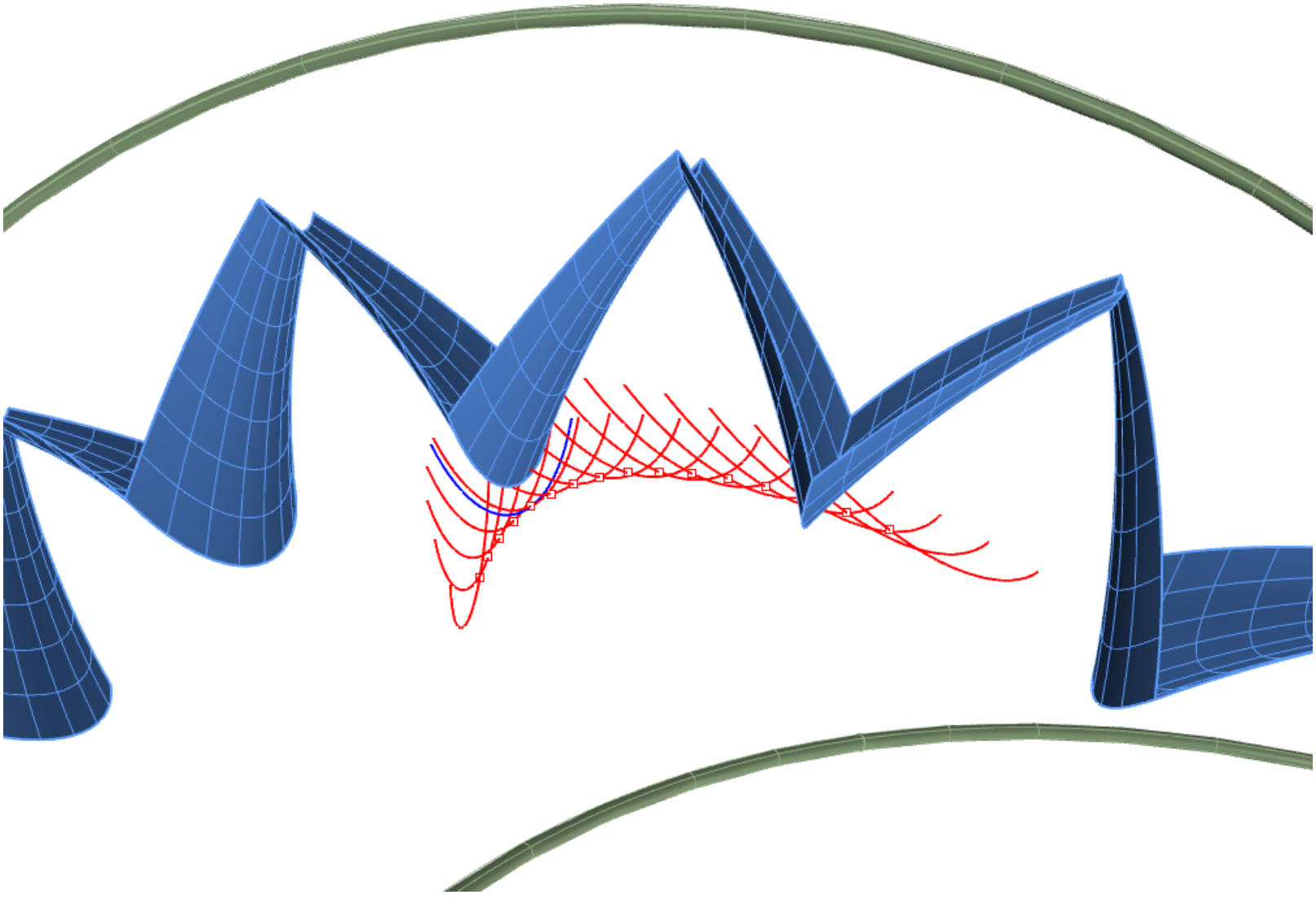

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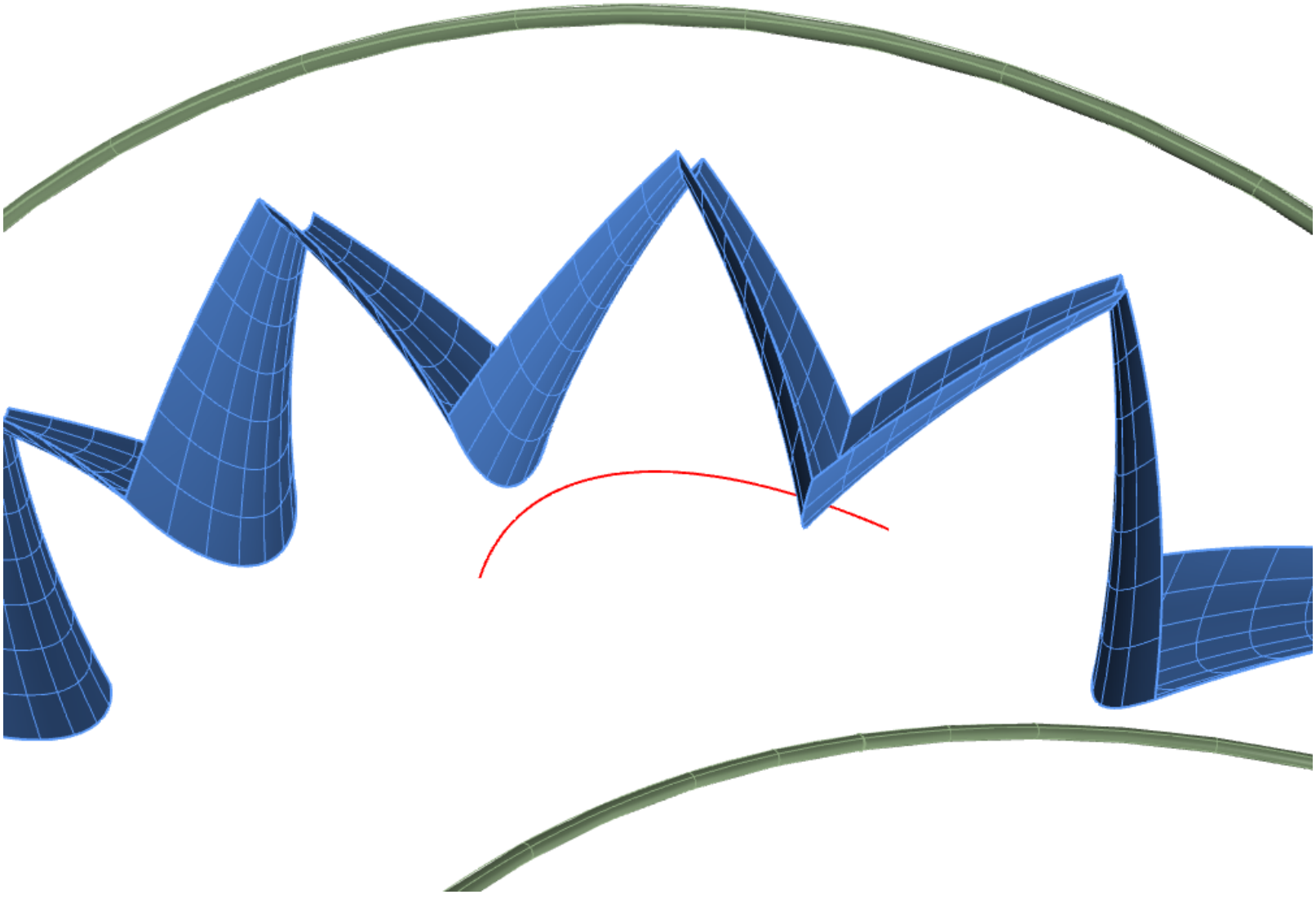

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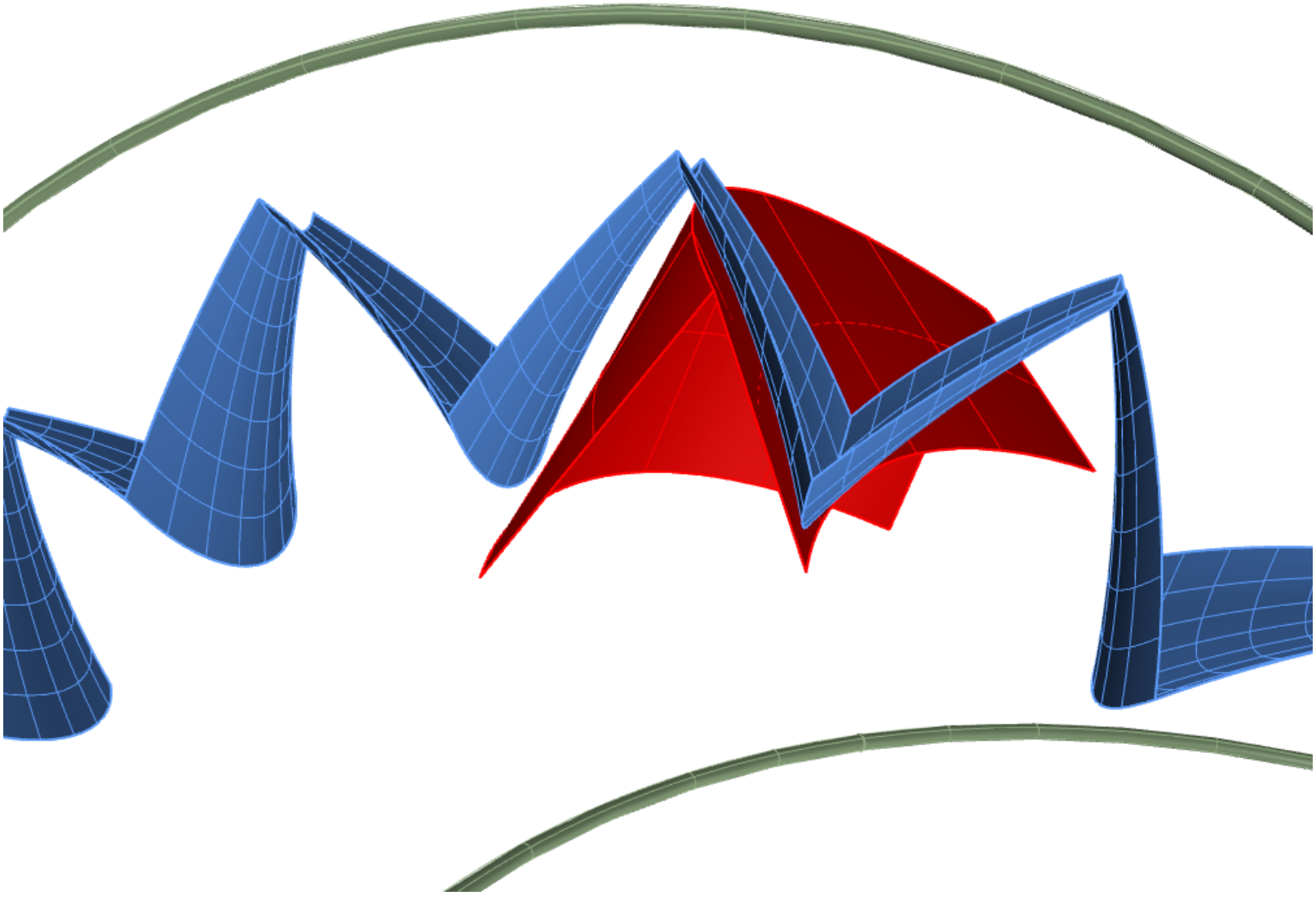

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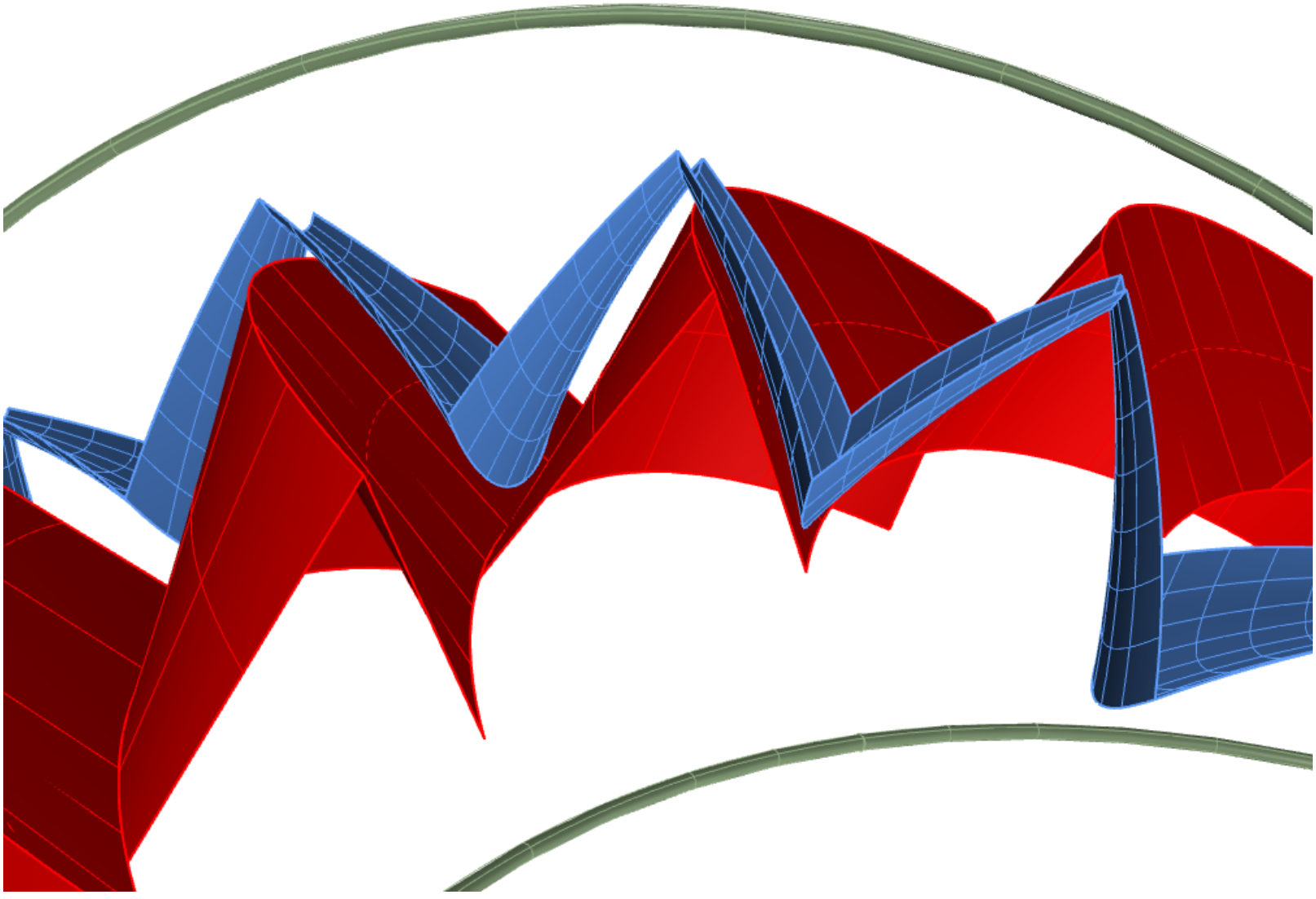

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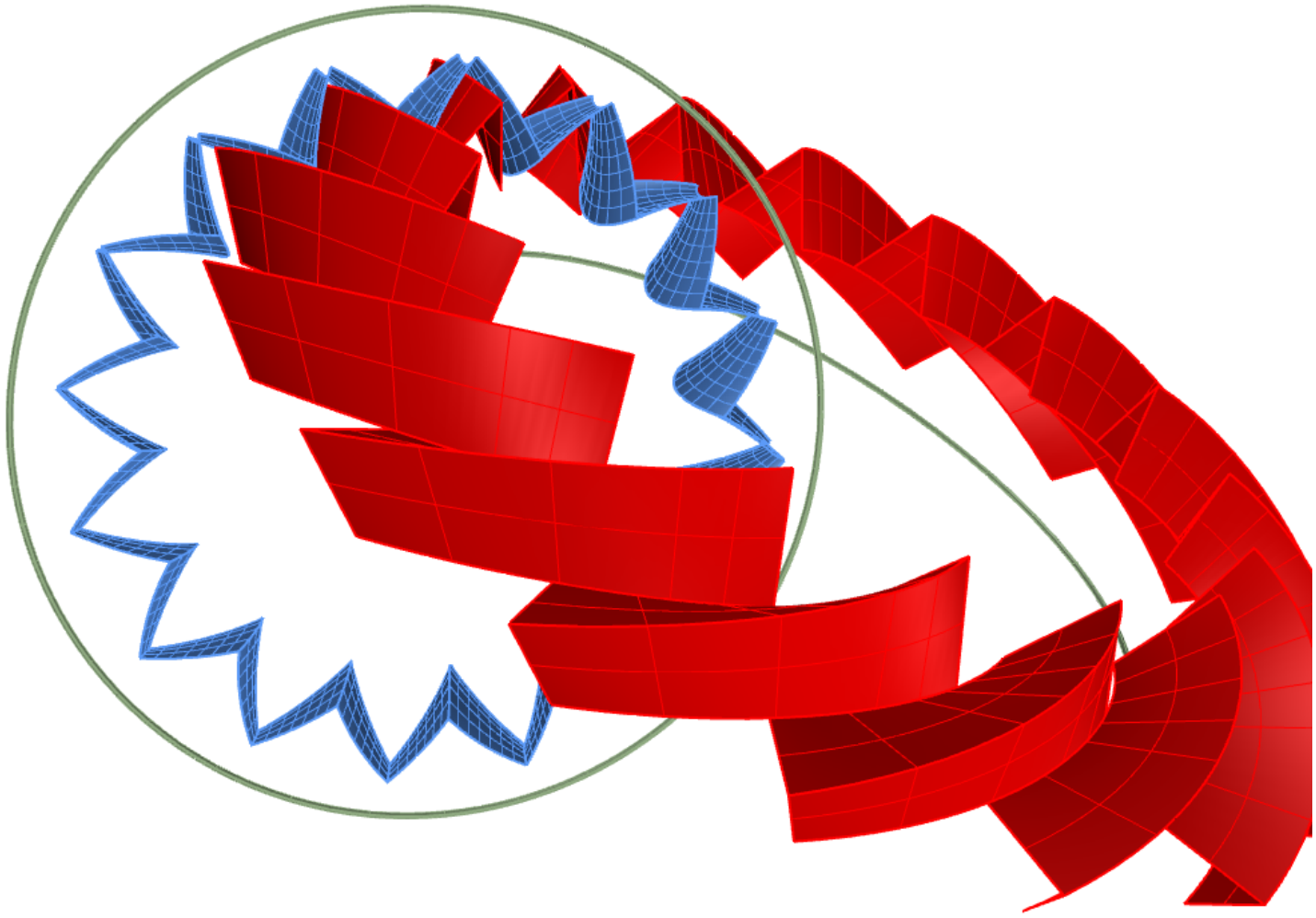
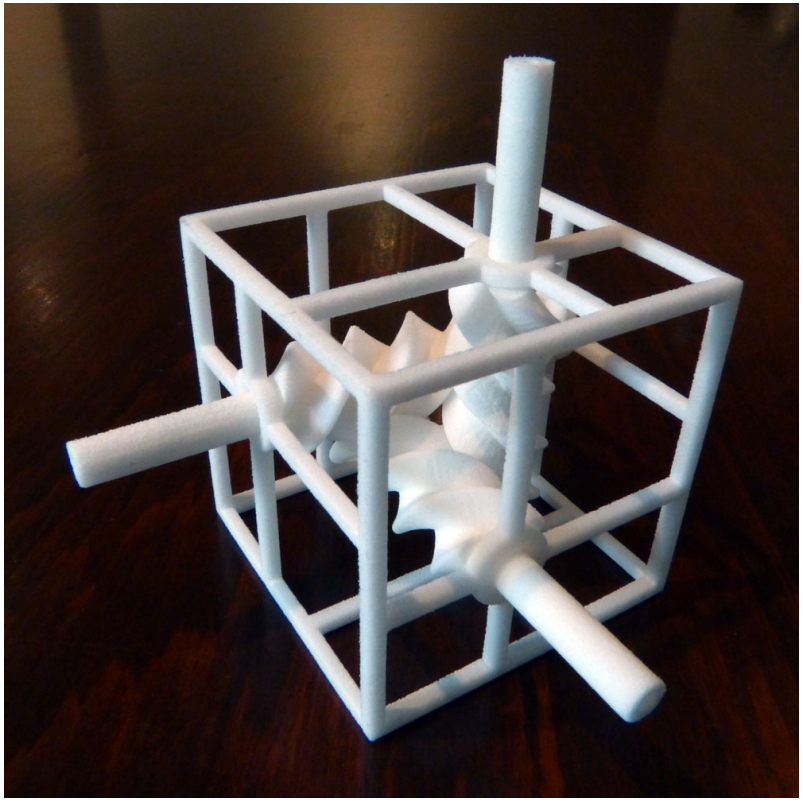
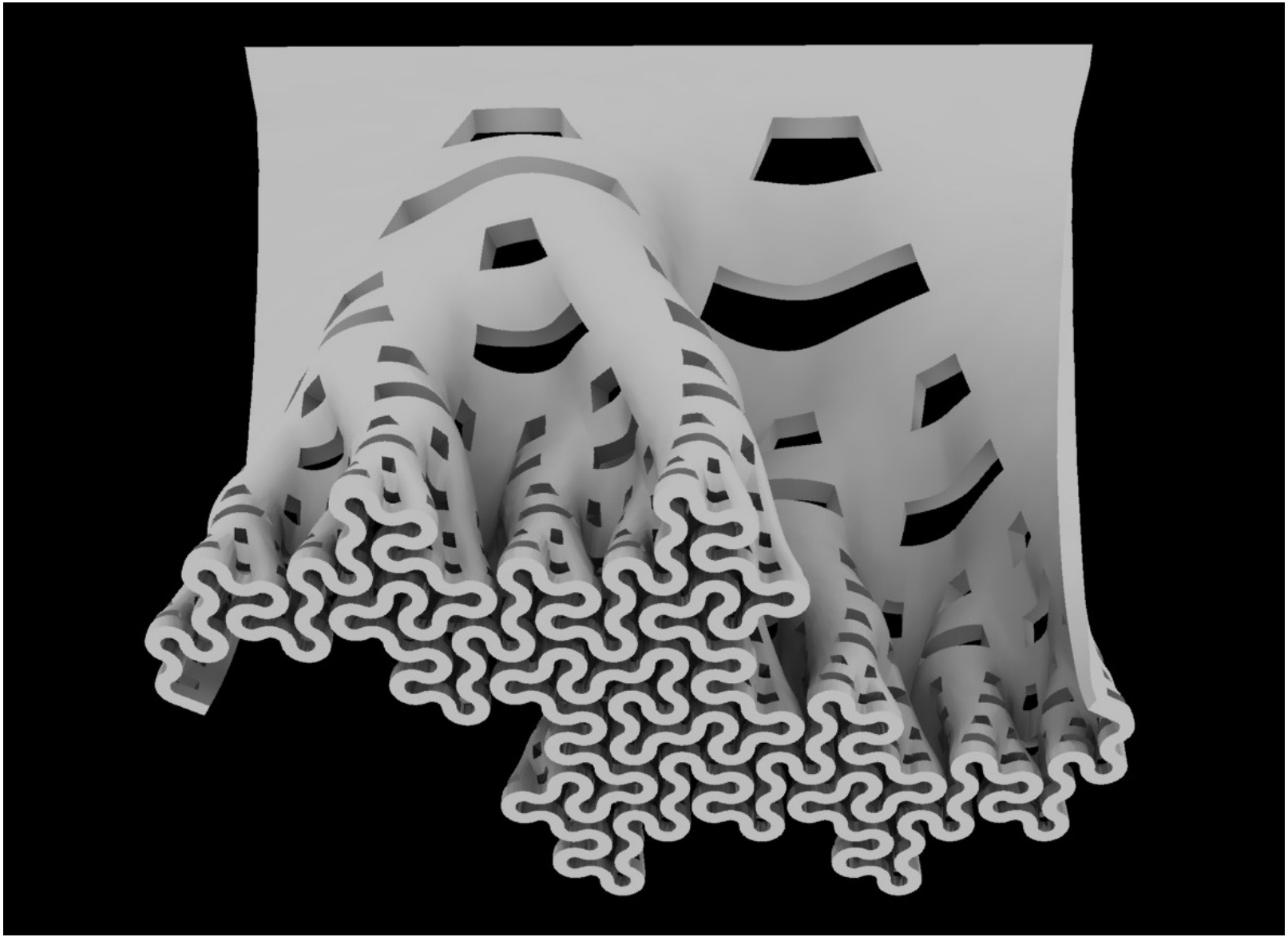
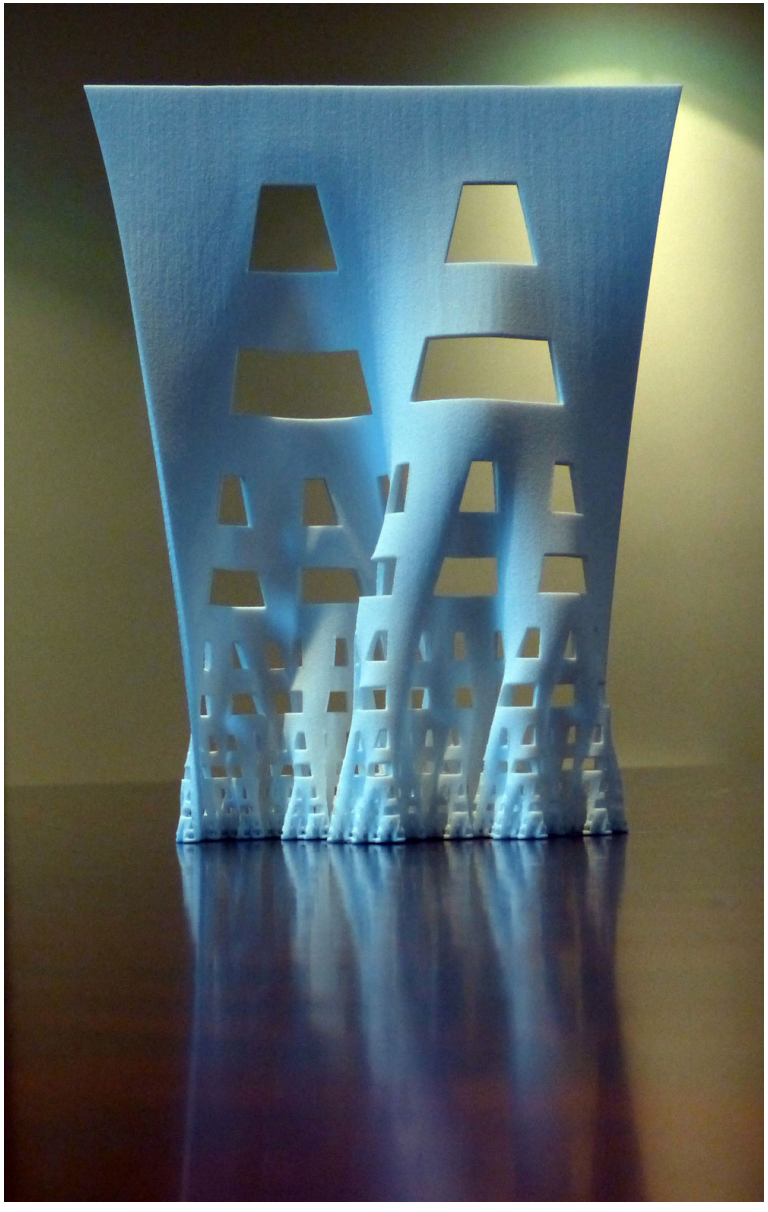


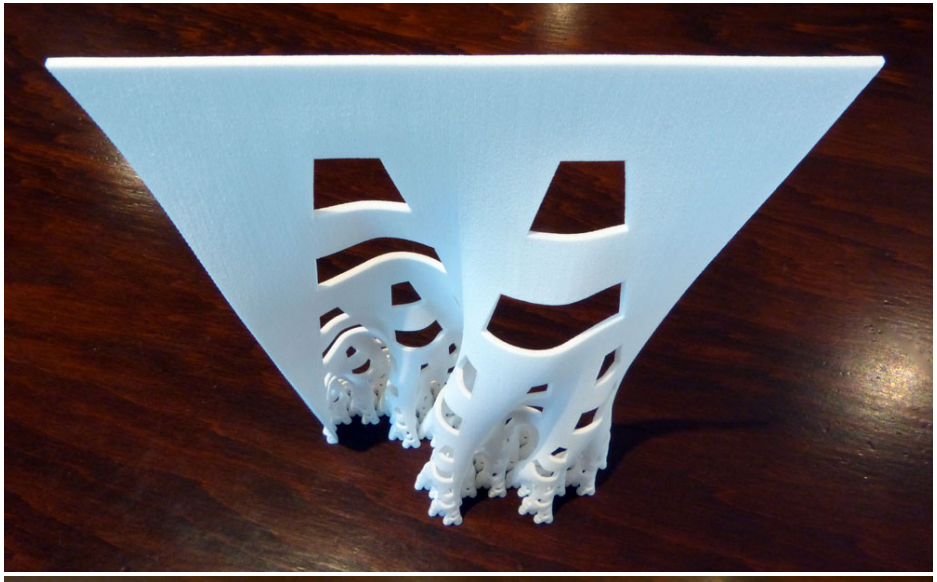
Alternative solutions

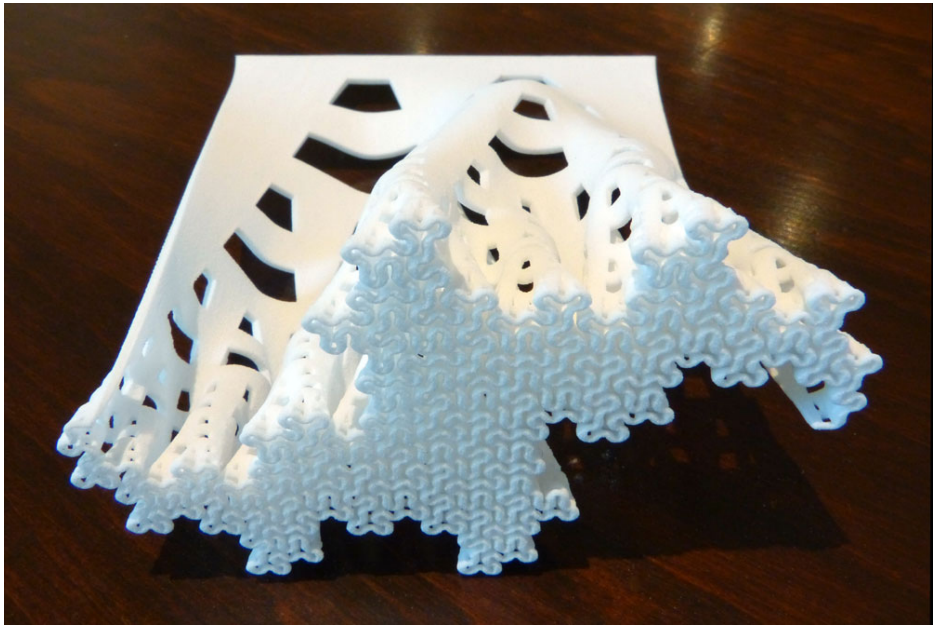


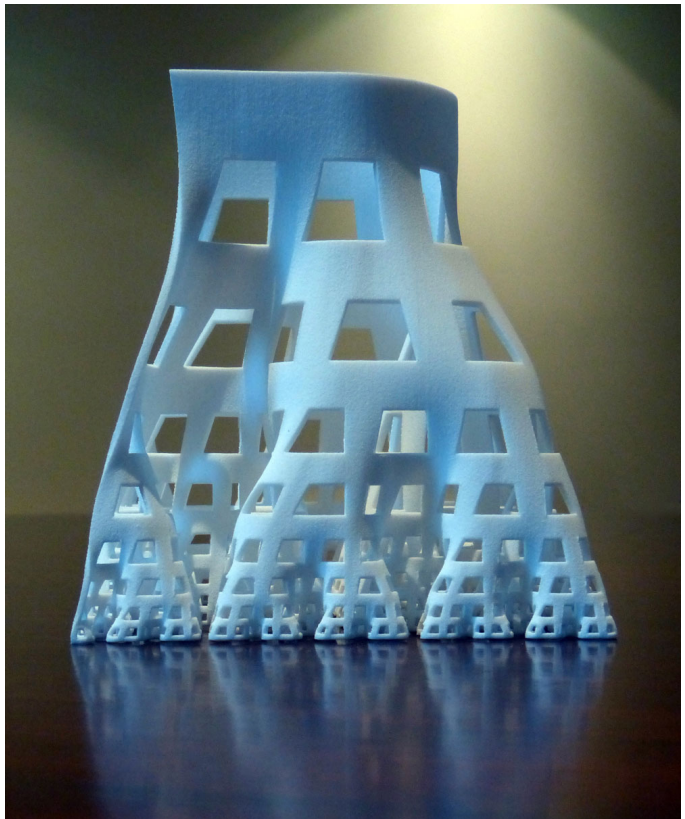
Developing fractal curves
(joint work with Geoffrey Irving)

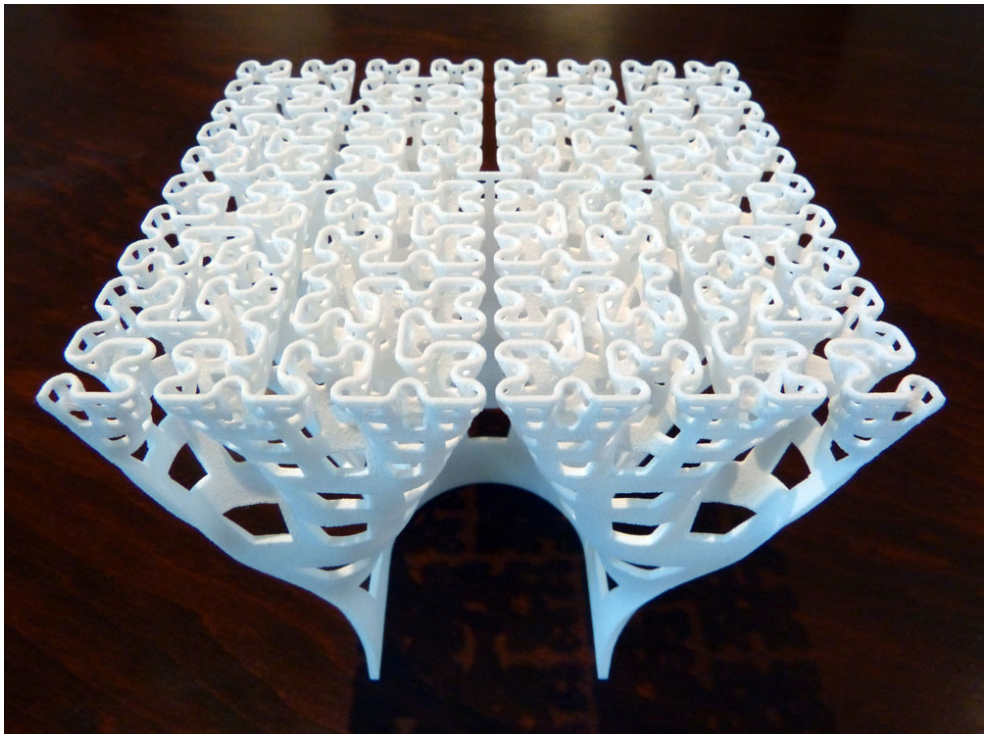


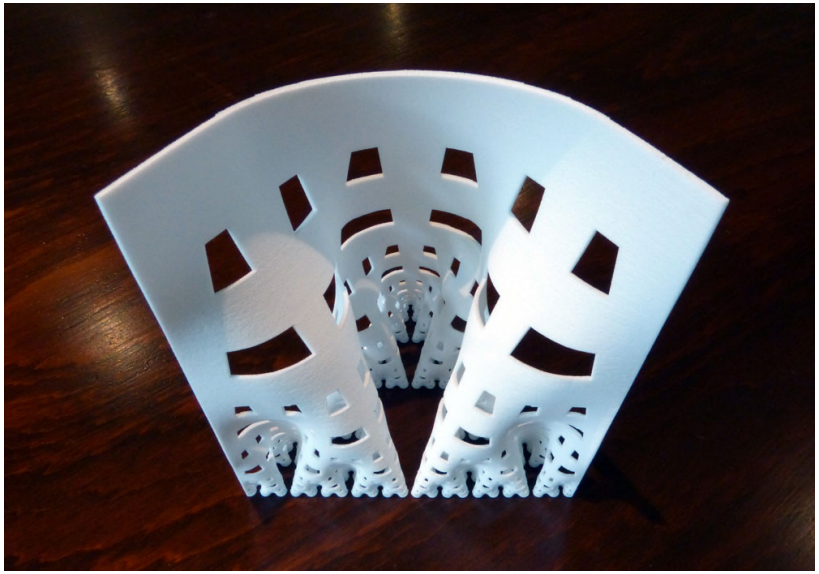


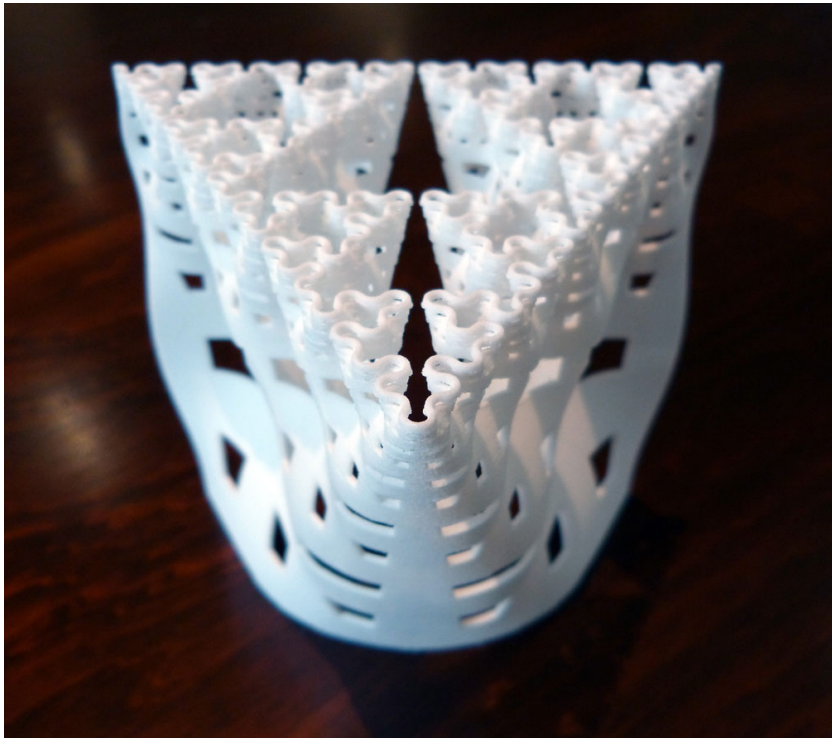


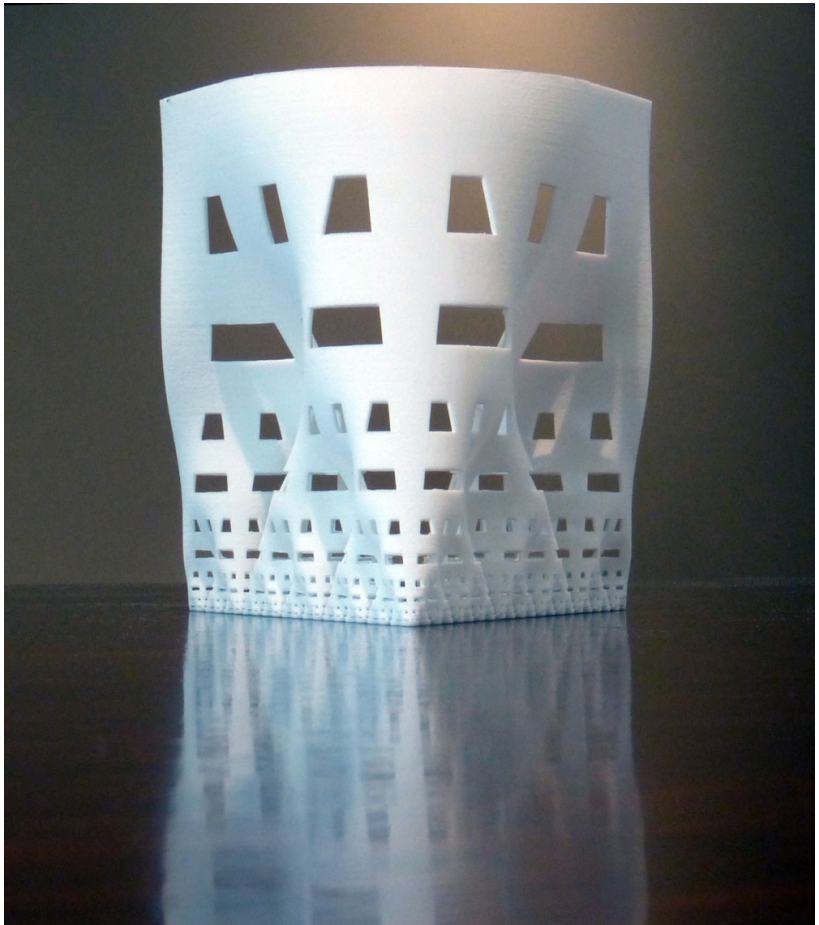


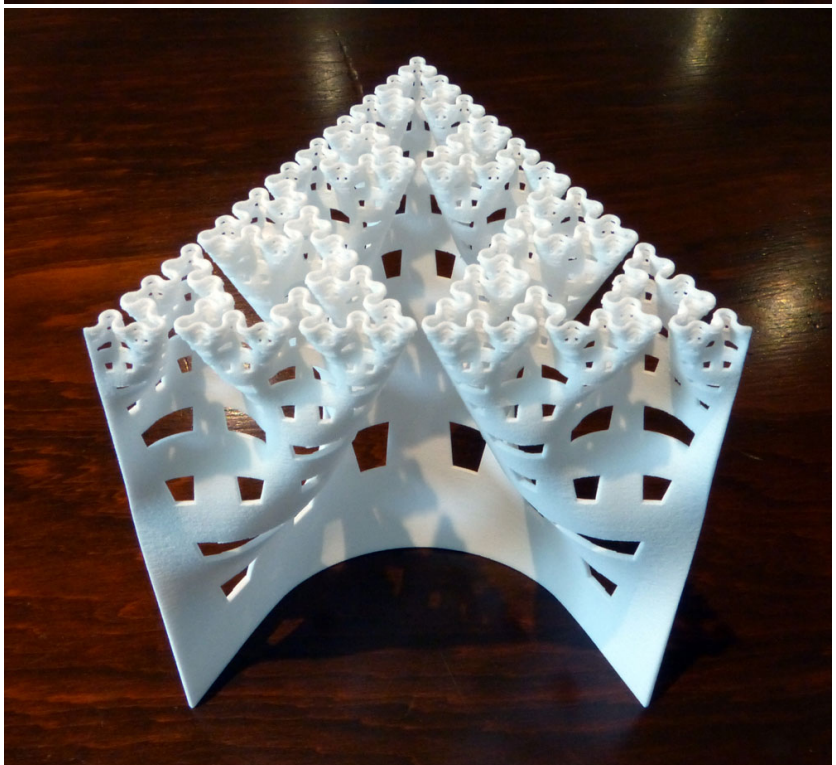


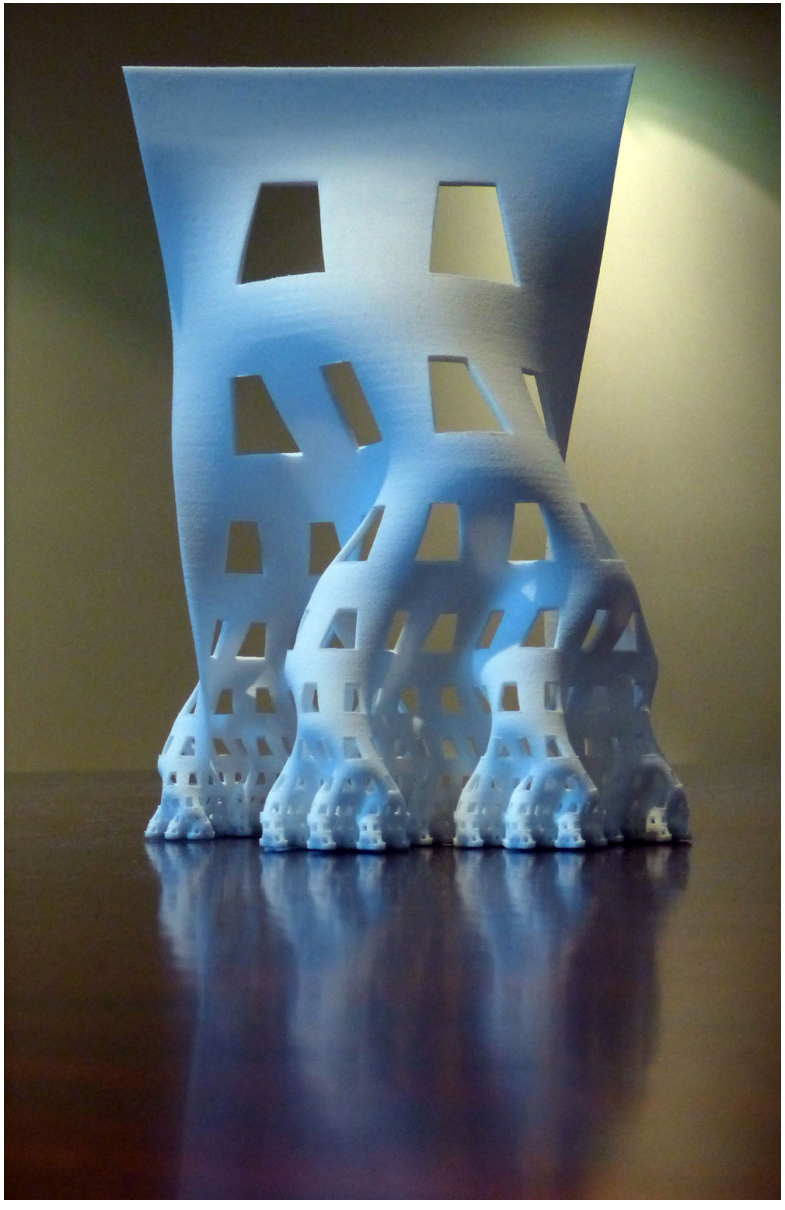


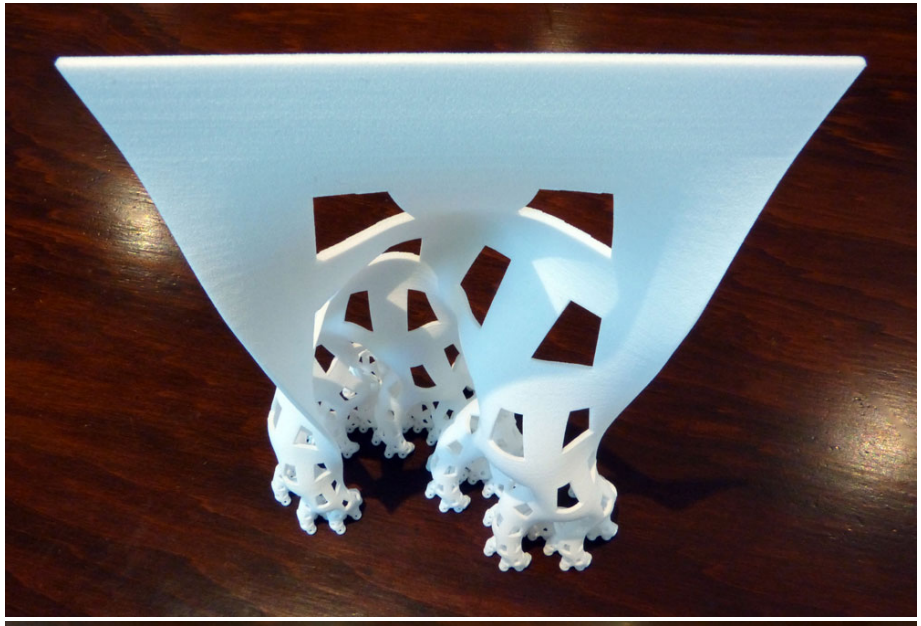


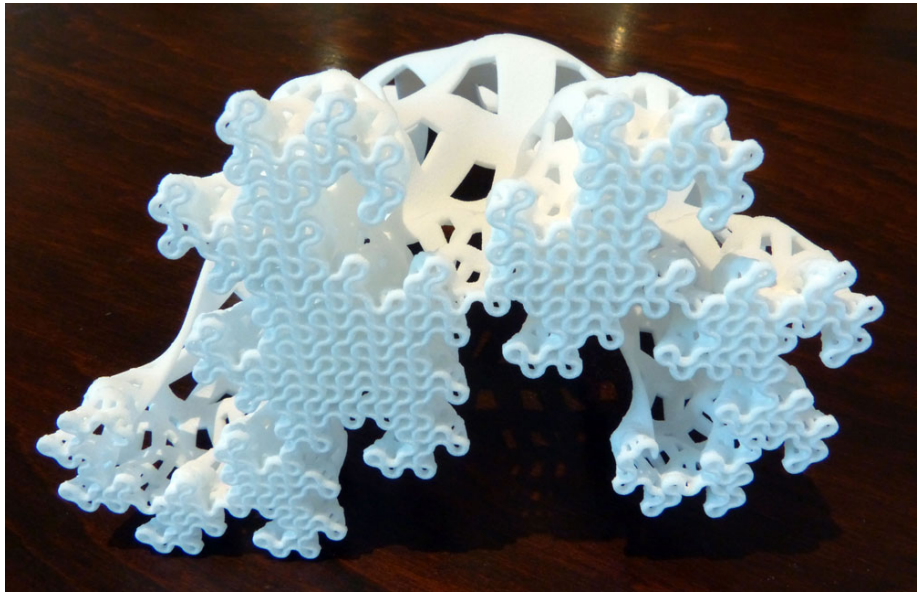


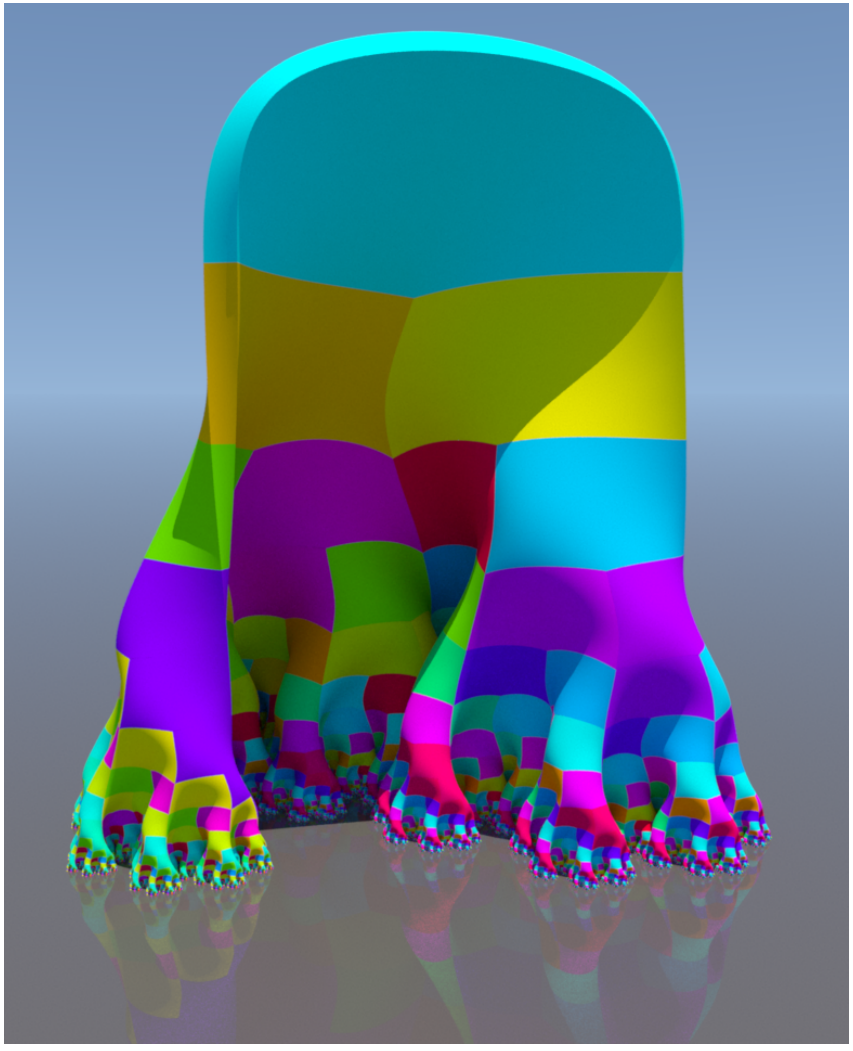


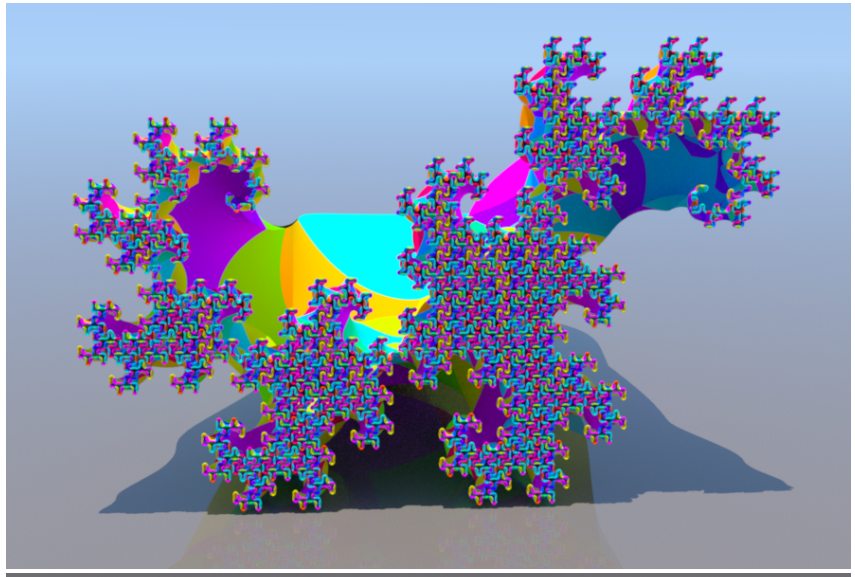


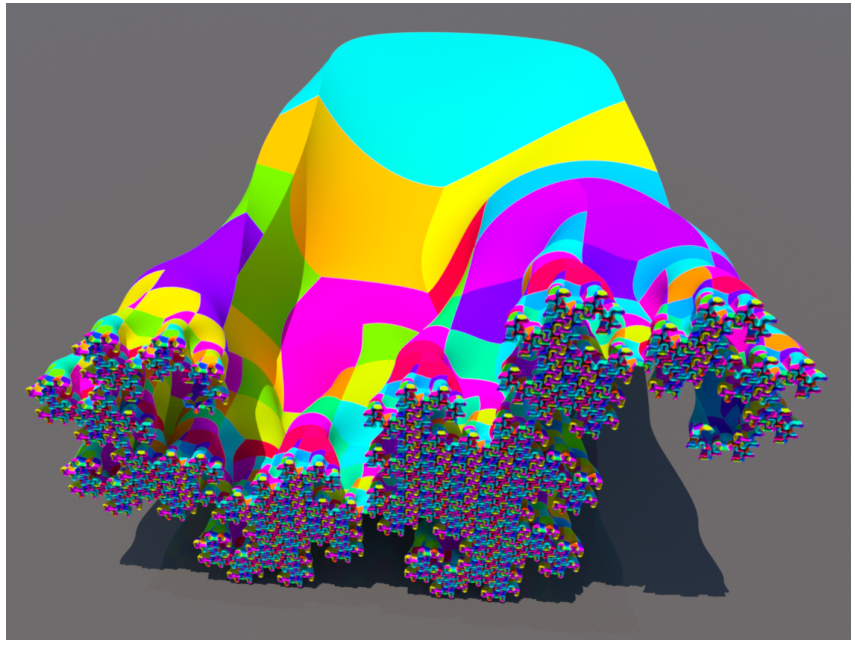




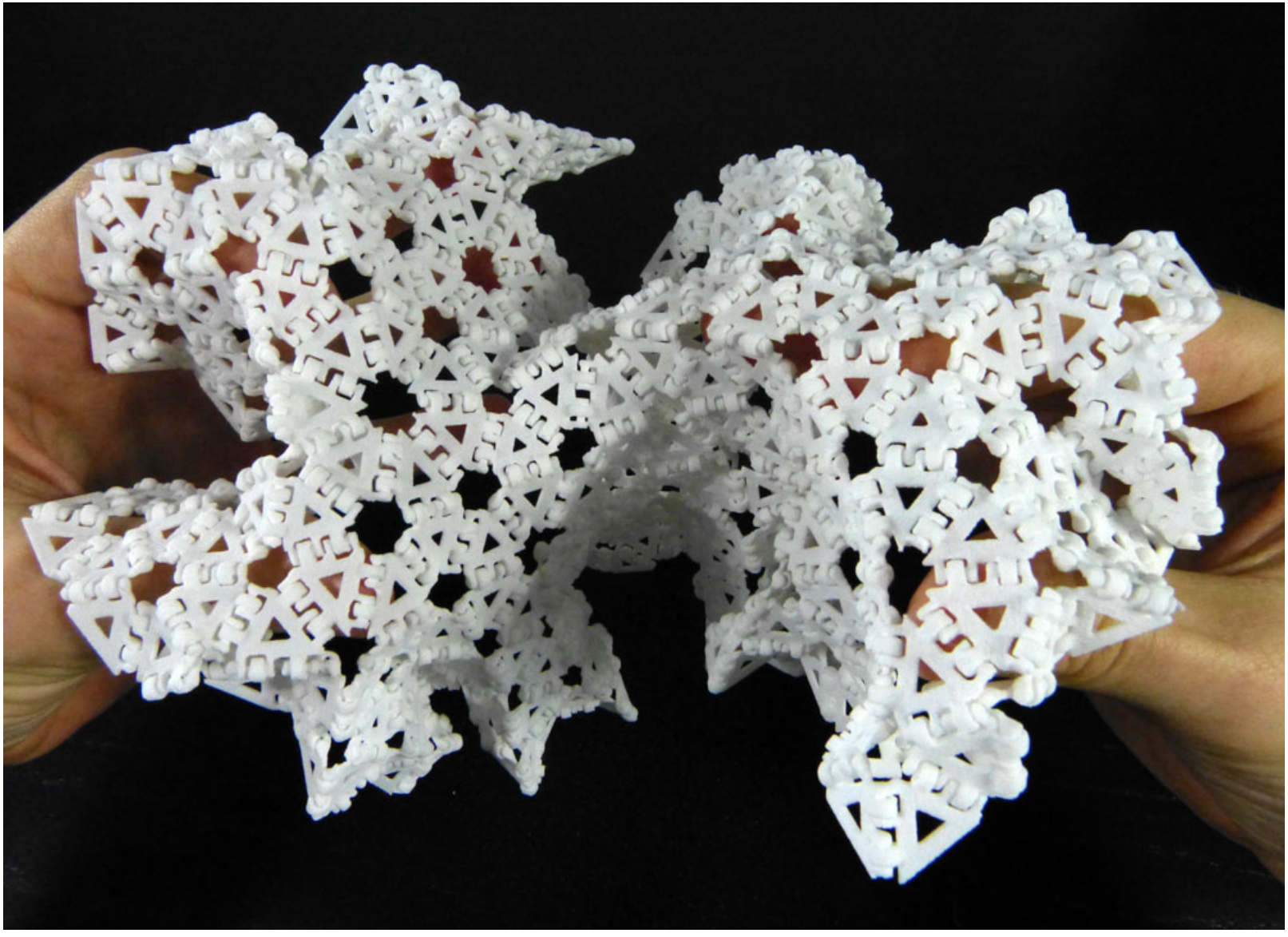


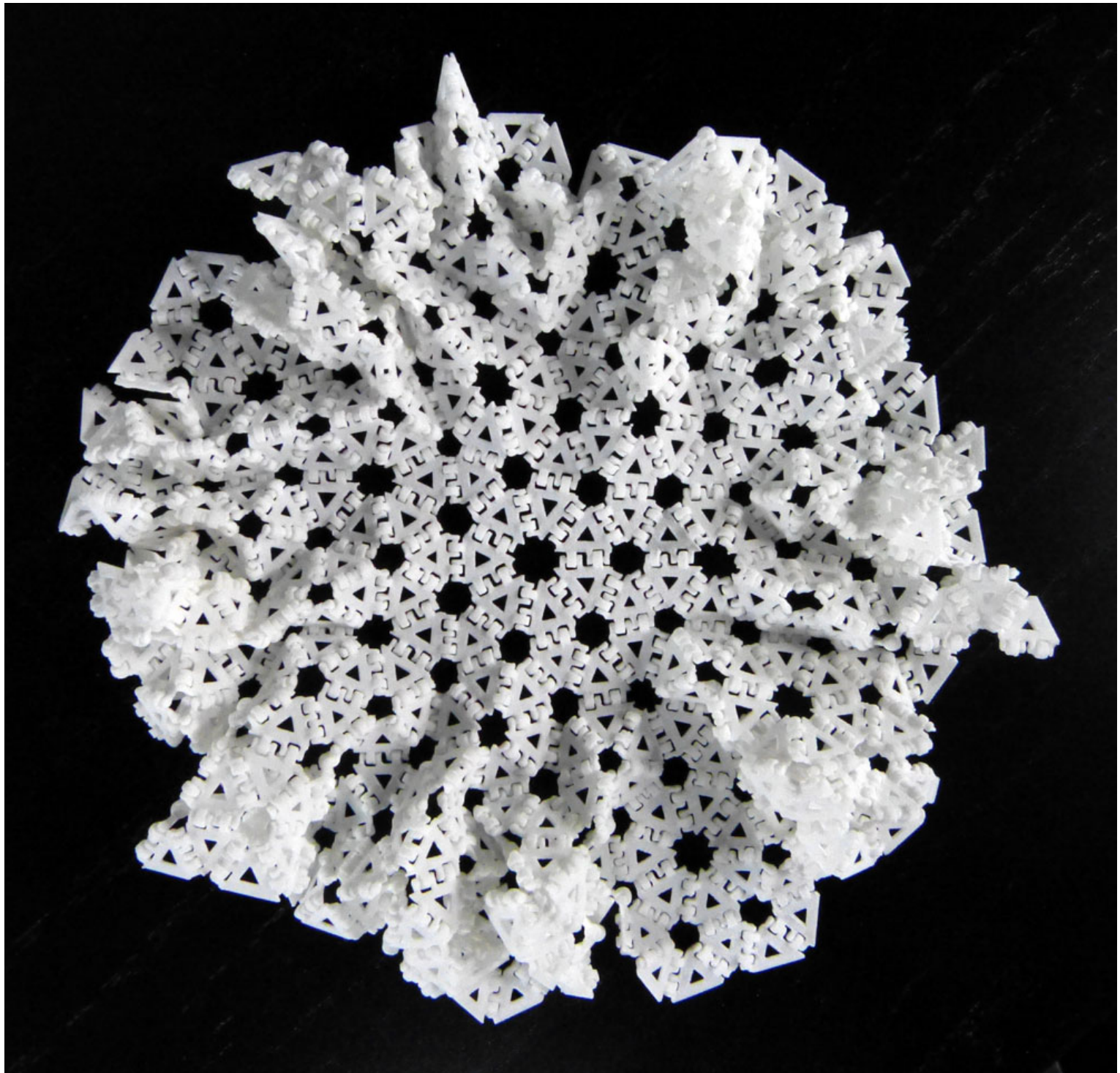


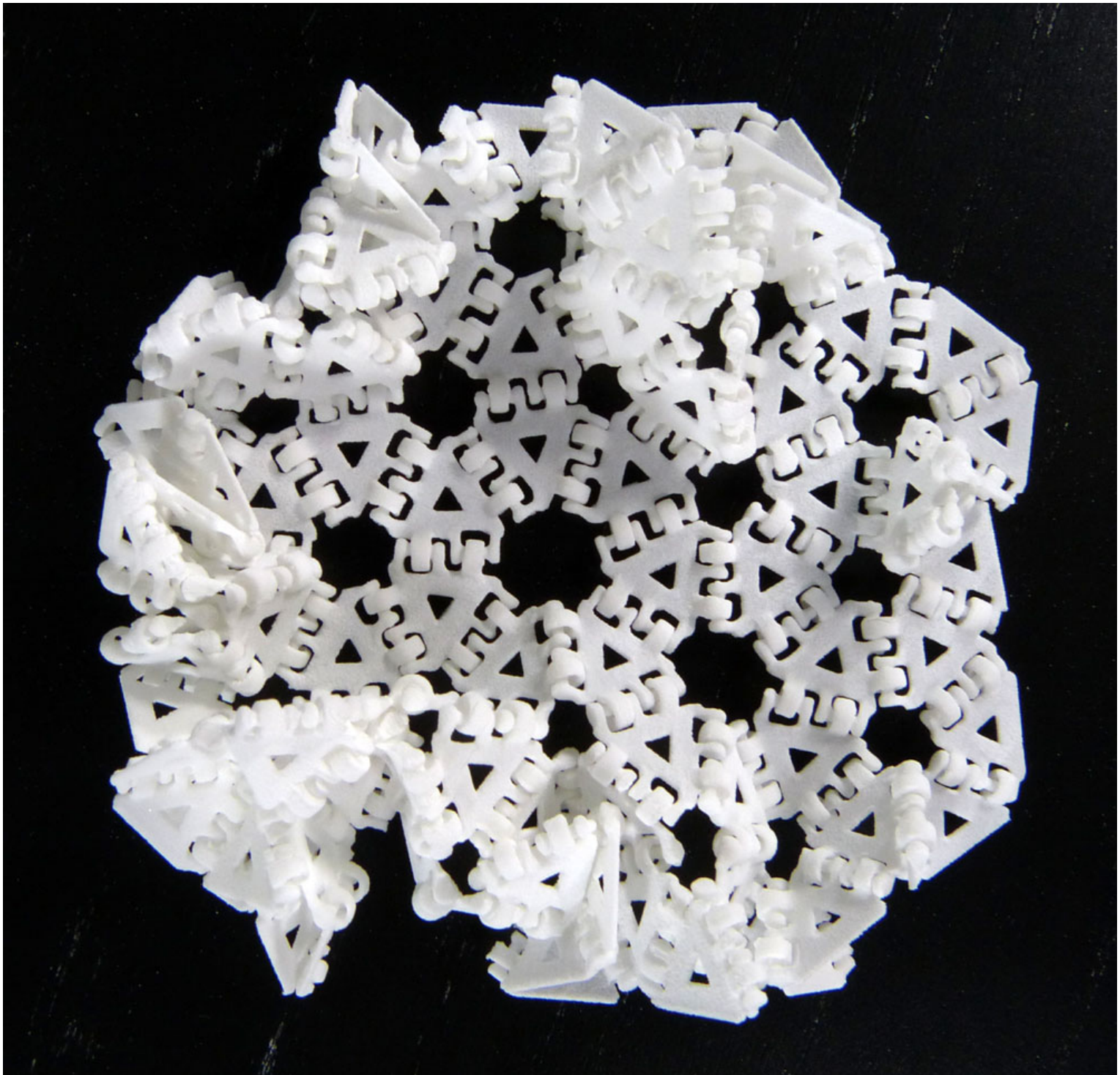


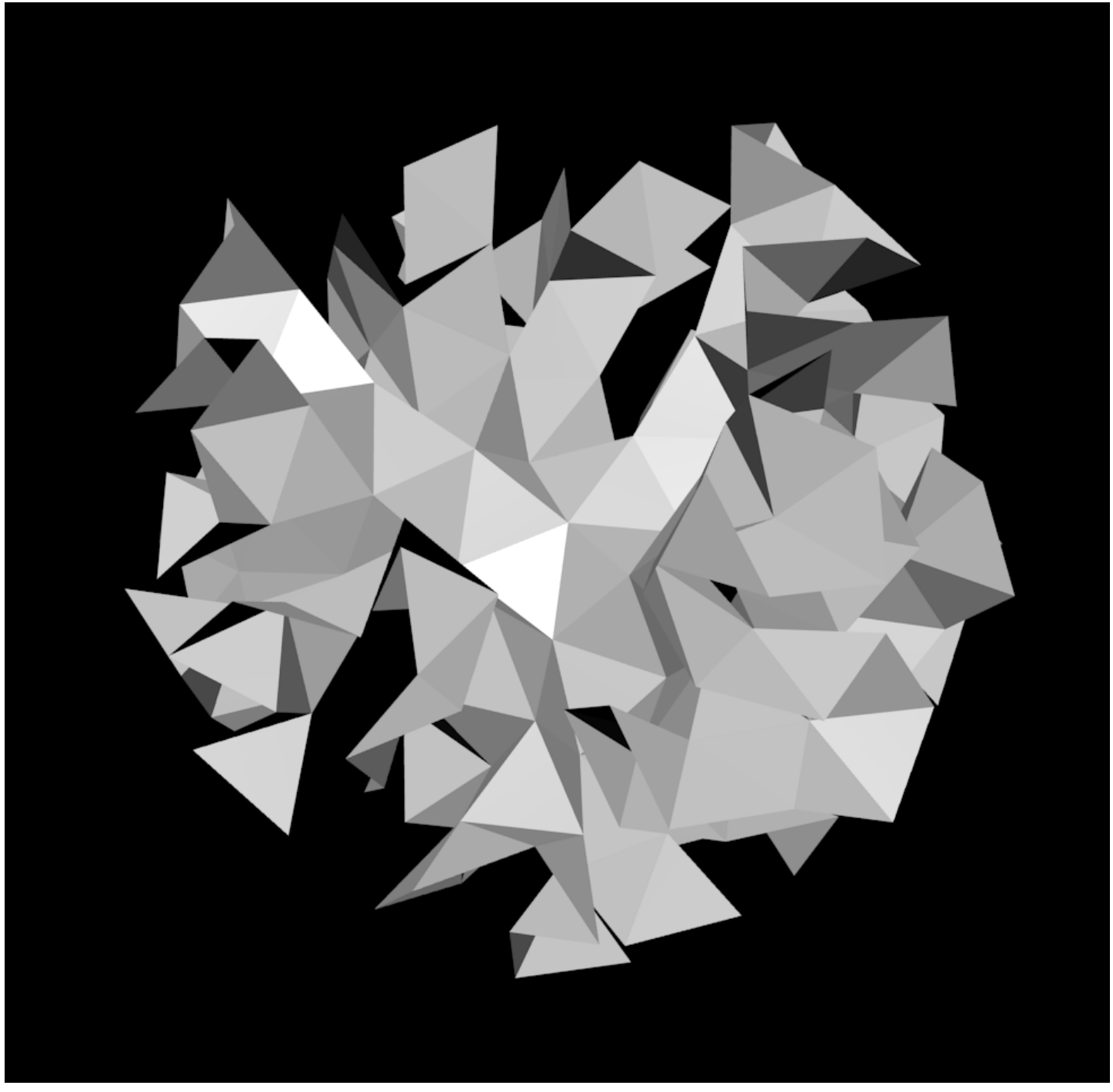


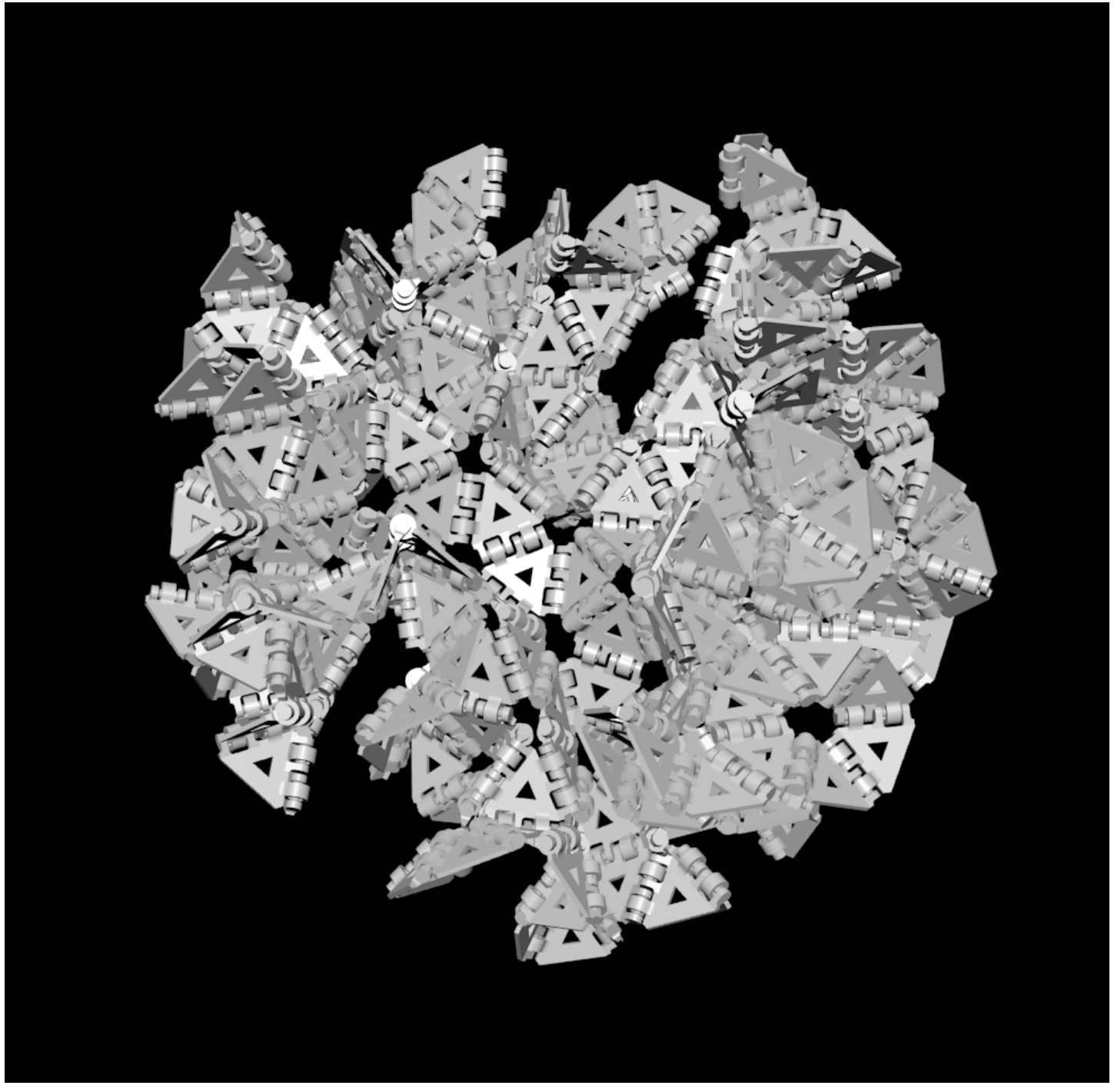
Hinged negatively curved surfaces (joint work with Geoffrey Irving)



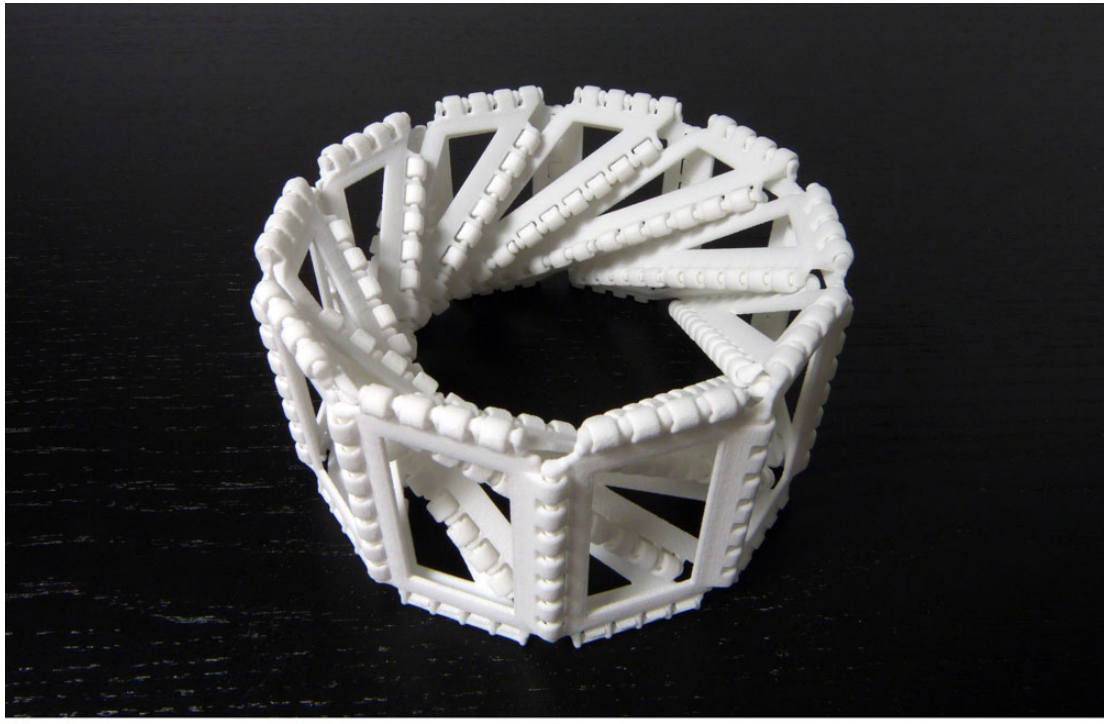


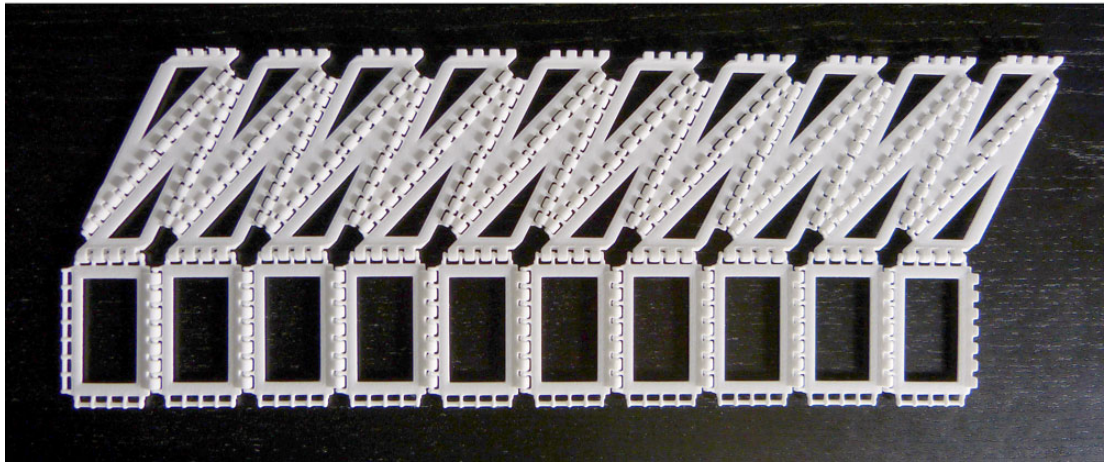






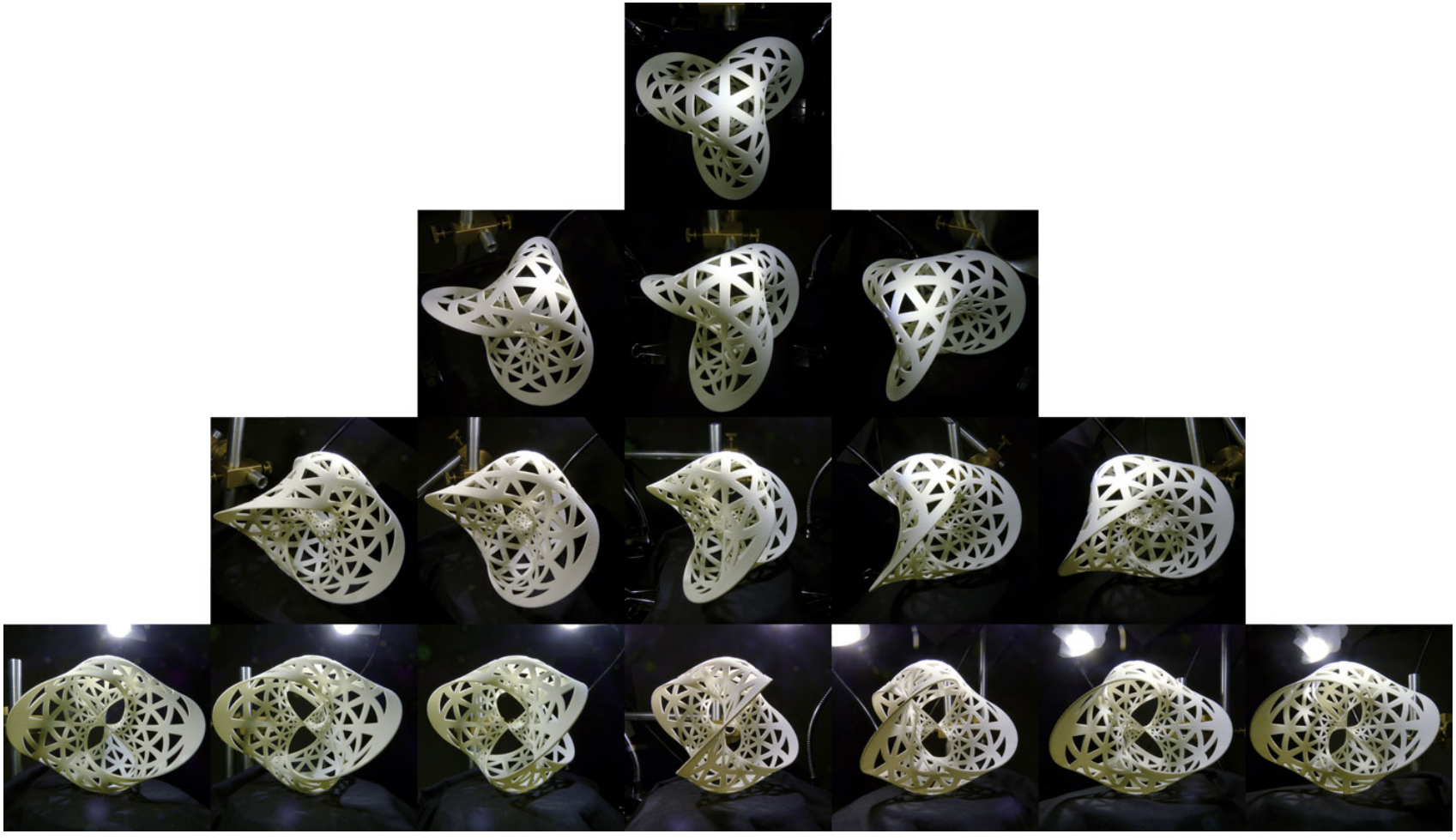
More hinges



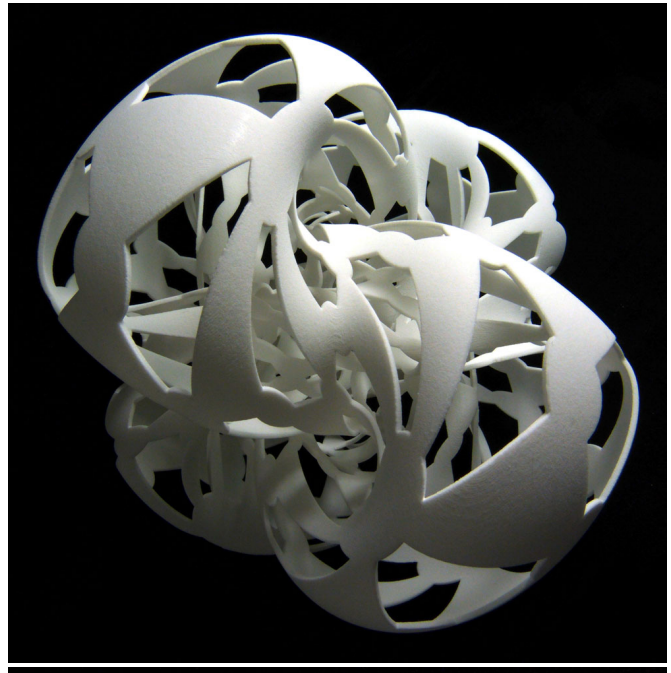


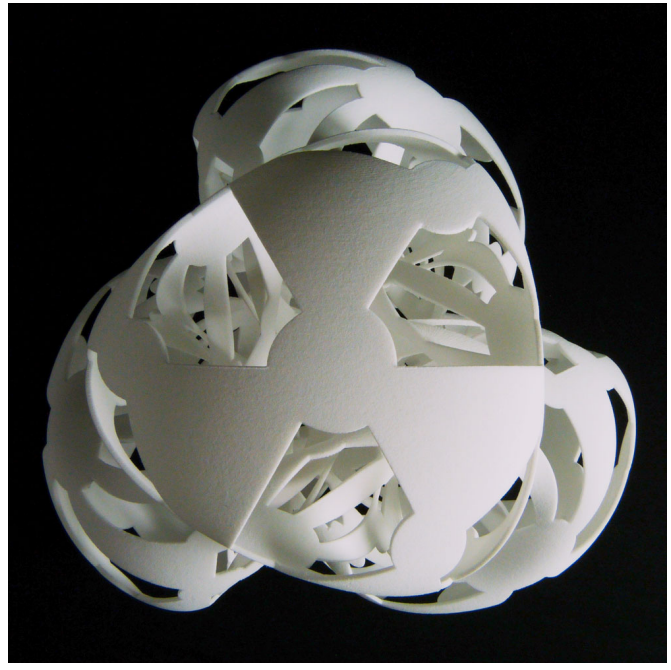
Topology joke (joint work with Keenan Crane)

Seifert surface on the (3,3) torus link (joint work with Saul Schleimer)



Klein quartic (joint work with Saul Schleimer)

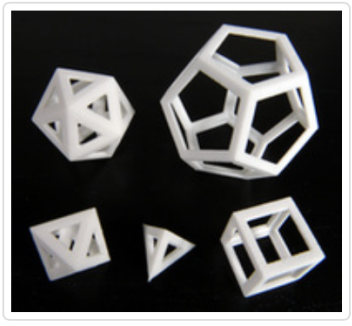





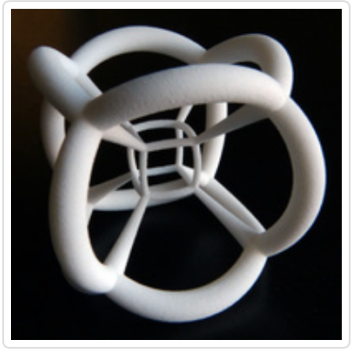
Book: Visualizing Mathematics with 3D Printing

1. Symmetry

2. Polyhedra



3. Four-dimensional space



4. Tilings and curvature

5. Knots

6. Surfaces

7. Menagerie

http://3dprintmath.com

References

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- *The Quaternion Group as a Symmetry Group*, Vi Hart and Henry Segerman, 2014.
- *Developing fractal curves*, Geoffrey Irving and Henry Segerman, 2013.
- *Sculptures in S^3*, Saul Schleimer and Henry Segerman, 2012.
- *Triple gear*, Saul Schleimer and Henry Segerman, 2013.
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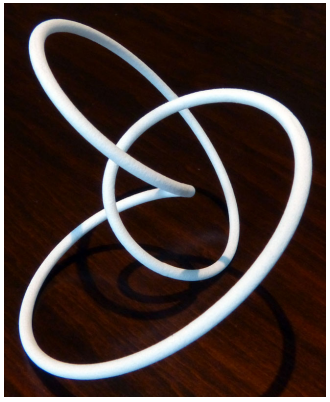
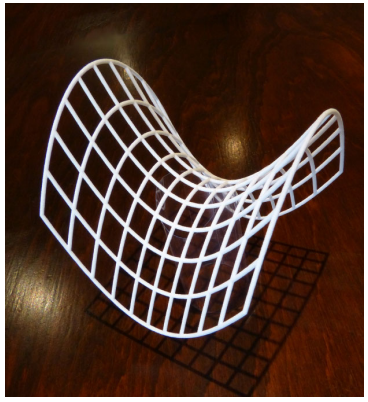
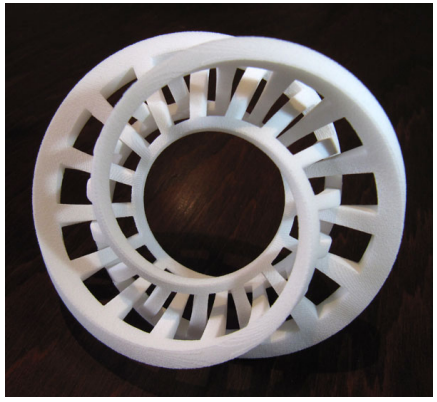
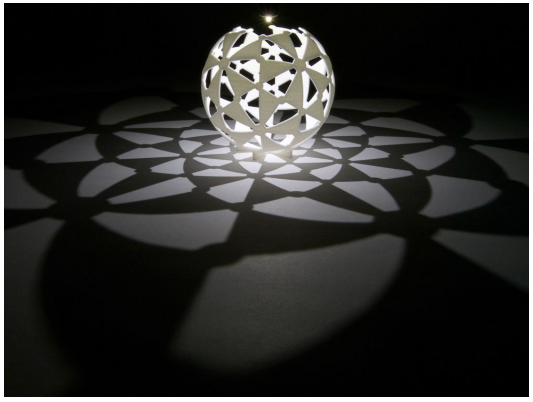
Thanks!

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