Math 109: Winter 2014
Midterm 2

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and explain your reasoning. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [5 + 15 points] Let X be a set and let n be a positive integer. (a) Carefully and formally define what it means to say "$|X| = n$". (Hint: Your answer should involve a function.) (b) If $|X| = n$, Y is a set, $f : X \to Y$ is a function, and G_f denotes the graph of f, prove that $|G_f| = n$.

Problem 2: [20 points] Let S be a sphere and let P_1, P_2, P_3, P_4, and P_5 be five points on S. Prove that there is a closed hemisphere H on S containing at least four of these points.

Problem 3: [15 points] Let X and Y be sets and let $f : X \to Y$ be a function. Suppose that there exists a function $g : Y \to X$ such that $g \circ f = I_X$. Prove that f is injective.

Problem 4: [20 points] Prove or disprove: For any sets X and Y, there exists a bijection between the sets $\mathcal{P}(X) \times \mathcal{P}(Y)$ and $\mathcal{P}(X \times Y)$.

Problem 5: [20 points] Let $\mathcal{F}$ be a family of real-valued functions on the open interval $(0, 1)$. The family $\mathcal{F}$ is called *equicontinuous* if for all $x \in (0, 1)$ and for all $\epsilon > 0$, there exists $\delta > 0$ such that for all $y \in (0, 1)$ with $|x - y| < \delta$, we have that $|f(x) - f(y)| < \epsilon$ for all $f \in \mathcal{F}$. State what it means for the family $\mathcal{F}$ to **not** be equicontinuous.