Name: ______________________________ **PID:** ________________

Math 109
Midterm Exam 2
May 25, 2007

Turn off and put away your cell phone.
You may use a calculator; but no other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	4	
2	6	
3	8	
4	6	
5	6	
Σ	30	

1. (4 points) Using the Euclidean algorithm, find $\gcd(3589, 2627)$ and find integers m and n such that $\gcd(3589, 2627) = 3589\, m + 2627\, n$.

2. (6 points) Let n be a positive integer. Prove that 7 divides $6^n + 1$ if and only if n is odd. [Hint: $6 \equiv -1 \mod 7$.]

3. (8 points) Define $\sim$ on $\mathbb{R} - \{0\}$ by $x \sim y$ if and only if $xy > 0$.

(a) Prove that $\sim$ is an equivalence relation on $\mathbb{R} - \{0\}$.

(b) Determine the equivalence classes on $\mathbb{R} - \{0\}$ corresponding to $\sim$, and find a complete set of equivalence class representatives (i.e., list one equivalence class representative from each equivalence class).

(c) Is $\sim$ is an equivalence relation on $\mathbb{R}$? Justify your answer.

4. (6 points) Let $[a] \in \mathbb{Z}_m$. Prove that there is an integer b such that $[a] \cdot [b] = [1]$ if and only if $\gcd(a, m) = 1$.

5. (6 points) Let $X = \{a, b, c\}$ and $Y = \{d, e\}$.

(a) What is the cardinality of the Cartesian product $X \times Y$?

(b) Write down an explicit bijection $f : \mathbb{N}_n \to X \times Y$, where $n = |X \times Y|$.