

Math 109: Winter 2014
Midterm 1

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and justify your answers. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [20 pts.] Let P and Q be statements. Use truth tables to show that the statements ‘not($P \Rightarrow Q$)’ and ‘ P and (not Q)’ are equivalent.

Problem 2: [20 pts.] Let x and y be positive real numbers with $x \neq y$. Prove that $\frac{x}{y} + \frac{y}{x} > 2$. (Hint: Try working backwards.)

Problem 3: [15 pts.] Let a , b , and c be integers. Prove that if a divides b and b divides c , then a divides c .

Problem 4: [20 pts.] Let n be a positive integer. Prove that n is even if and only if n^2 is even.

Problem 5: [20 pts.] For all positive integers n , prove that $2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 2$.