Name: ______________________________ **PID:** ________________

Math 109
Midterm Exam 1
April 27, 2007

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
Σ	30	

1. (6 points) Let D be a division ring. Consider the following statement:

   - D is a field is a necessary condition for D to be finite.

   For purposes of this question, it is not necessary to know what "division ring", "field", or "finite" mean.

   (a) Write the contrapositive of the statement.

   (b) Write the converse of the statement.

   (c) Write the negation of the statement.

2. (6 points) Consider the following proof that 1 is the smallest positive real number.

Let x be the smallest positive real number. Clearly, $x \leq 1$. On the other hand, $x^2 \leq x$. Thus, $x(x-1) = x^2 - x \geq 0$. Therefore, $x \geq 1$ and it follows that $x = 1$.

(a) What is wrong with the proof?

(b) What correct statement does this proof actually prove? What type of proof is it?

3. (6 points) Recall the following definitions for any sets P, Q, R and S:

(i) $x \in P \cap Q$ if and only if $x \in P$ and $x \in Q$.

(ii) $(x, y) \in S \times T$ if and only if $x \in S$ and $y \in T$.

Let A, B and C be sets. Prove directly using the above definitions that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

(Note: neither a truth table nor a Venn diagram meets the requirements of this problem and so will not earn any credit.)

4. (6 points) Prove by induction that for every positive integer n,

$$\sum_{k=1}^{n} \left(3k^2 - 3k + 1\right) = n^3.$$

5. (6 points) Suppose that $f : X \to Y$ and $g : Y \to Z$ are injections. Prove that $g \circ f : X \to Z$ is an injection.