Math 109 – Homework Assignment 6
Due Friday, May 18

1. Exercise 12.4. Three people each select a main dish from a menu of five items. How many choices are possible (i) if we record who selected which dish (as the waiter should), and (ii) if we ignore who selected which dish (as the chef could)?

2. Exercise 12.6. Prove that the product of any n consecutive positive integers is divisible by $n!$.

3. Problem III.11. Use the pigeonhole principle and proof by contradiction to prove Theorem 11.1.7: given non-empty finite sets X and Y with $|X| = |Y|$, a function $X \to Y$ is an injection if and only if it is a surjection.

4. Problem III.14. For $n \in \mathbb{N}$, suppose that $A \subseteq \mathbb{N}_{2n}$ and $|A| = n + 1$. Prove that A contains a pair of distinct integers a, b such that a divides b.

   [Let $f(a)$ be the greatest odd integer which divides a and apply the pigeonhole principle to f.]

5. Problem III.20. Use the pigeonhole principle to prove that, given ten distinct positive integers less than 107, there exist two disjoint subsets with the same sum.