Math 109 – Homework Assignment 1
Due Friday, April 13

Exercises can be found at the end of the corresponding chapter of the book, and solutions for each exercise can be found in the back of the book. Problems can be found at the end of the corresponding part of the book. There are no solutions in the back for these problems.

1. Exercise 1.1. Complete the truth table for "and":

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

2. Exercise 1.2 (i), (ii). Construct the truth table for the statements

   (i) $\neg(P \wedge Q)$;

   (ii) $(\neg P) \vee (\neg Q)$.

P	Q	$\neg(P \wedge Q)$
T	T	
T	F	
F	T	
F	F	

P	Q	$(\neg P) \vee (\neg Q)$
T	T	
T	F	
F	T	
F	F	

   Conclude that the statements "$\neg(P \wedge Q)$" and "$(\neg P) \vee (\neg Q)$" are equivalent.

3. (cf. Exercise 1.4.) Consider the following statement.

   (i) All rich people are happy.

   Which of the following statements is the negation of the above statement?

   (i) All rich people are sad.

   (ii) There is no rich person who is happy.

   (iii) Some rich person is sad.

   (iv) Some rich person is not happy.

   (v) All people who are happy are rich.

   (vi) All people who are not happy are not rich.

   Can you find any statements in this list that which have the same meaning as the original statement (i)?

4. Find the negation of the statement "All prime numbers are odd."
   Note: While it would be a correct answer, you are not allowed to just write "It is not true that all prime numbers are odd." In other words, you need to "simplify" your answer.

5. Exercise 2.3. Complete the truth table for $P \Leftrightarrow Q$ using the tables for "$\Rightarrow$" and "and".

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
T	T			
T	F			
F	T			
F	F			

6. Exercise 2.5. By using truth tables prove that, for all statements P and Q,

   (i) the statements "$P \Rightarrow Q$" and "$(\neg Q) \Rightarrow (\neg P)$" are equivalent,

   (ii) the statements "$P \vee Q$" and "$(\neg P) \Rightarrow Q$" are equivalent.

   Remark: The statement $(\neg Q) \Rightarrow (\neg P)$ is called the *contrapositive* of the statement $P \Rightarrow Q$.

7. Exercise 3.2. Prove that for all integers a, b, and c,

$$(a \text{ divides } b) \text{ and } (b \text{ divides } c) \Rightarrow a \text{ divides } c.$$

8. Exercise 3.7. Proposition 3.2.1 states that $a < b$ is a sufficient condition for $4ab < (a+b)^2$. Is this condition also necessary? If so, prove it. If not, find a necessary and sufficient condition.

9. Problem I.4. Prove the following statements concerning positive integers a, b, and c.

   (i) $(a$ divides $b)$ and $(a$ divides $c) \Rightarrow a$ divides $b + c$.

   (ii) $(a$ divides $b)$ or $(a$ divides $c) \Rightarrow a$ divides bc.

10. Problem I.5. Which of the following conditions are *necessary* for the positive integer n to be divisible by 6 (proofs not necessary)?

    (i) 3 divides n.

    (ii) 9 divides n.

    (iii) 12 divides n.

    (iv) $n = 12$.

    (v) 6 divides n^2.

    (vi) 2 divides n and 3 divides n.

    (vii) 2 divides n or 3 divides n.

    Which of these conditions are *sufficient*?