

Math 109: Winter 2014
Final Exam

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and explain your reasoning. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [5 points] Construct a truth table for the statement “[not(P or Q)] $\Rightarrow R$ ”.

Problem 2: [10 points] Prove that for all $n \in \mathbb{Z}^+$, we have $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 3: [10 points] Prove that two different people on Facebook have the same number of friends.

Problem 4: [10 points] A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is said to be *uniformly continuous* if for all $\epsilon > 0$ there exists $\delta > 0$ such that for all $x, y \in \mathbb{R}$ with $|x - y| < \delta$ we have that $|f(x) - f(y)| < \epsilon$. Carefully state what it means for f to **not** be uniformly continuous.

Problem 5: [5 + 10 points] (a) Define what it means for a set to be “denumerable”. (Your definition should involve a function.) (b) Is $[0, 1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$ denumerable? Prove your claim.

Problem 6: [10 points] Accurately state the Division Theorem.

Problem 7: [10 points] Let a be an integer. Prove that $5|a$ if and only if $5|a^2$.

Problem 8: [5 + 5 + 5 points] Give examples of sets X with binary relations \sim satisfying the specified conditions. You do not need to prove that your examples work.

- (1) The binary relation \sim on X is reflexive and symmetric, but not transitive.
- (2) The binary relation \sim on X is reflexive and transitive, but not symmetric.
- (3) The binary relation \sim on X is symmetric and transitive, but not reflexive.

Problem 9: [10 points] Prove or disprove: For any set X , there exists an injection $f : X \rightarrow \mathcal{P}(X)$.