Math 109 - L 1

Spring 2018

Instructor: Sean Curry

- Email: sncurry@ucsd.edu
- Office Hours: 3-4pm MWF

Piazza

Course webpage: math.ucsd.edu/~sncurry/109

↳ Syllabus ← please read!

- Textbook: An Intro. to Mathematical Reasoning, Peter J. Eccles
- Grading: Best of

(1) 20% HW + 20% for each midterm + 40% Final

or

(2) 20% HW + 20% best midterm + 40% final.

Main goals of this course:

- To introduce students to rigorous mathematics, i.e., to methods of mathematical reasoning by which we draw logical conclusions from clearly stated assumptions.
- To understand (rigorous) proofs, and to write clear proofs.
- Cover basic notions in math & apply what we have learned.

Requires problem solving →

Some topics we will cover:

- Propositions, truth tables, logical implications
- Proof techniques:
  - direct proof
  - proof by cases
  - proof by contradiction
  - proof by contrapositive
  - proof by induction.
- Set theory
- Countable vs uncountable sets

...

Chapter 1: The Language of Mathematics

1.1 Mathematical Statements

We are going to give the term "mathematical statement" a precise definition, which will include two kinds of statements:

(1) Propositions: A proposition is a sentence that is either true or false (but not both).

Examples:

(i) $1+2=3$. ← True.

(ii) $x+1>x$. ← True.

(iii) $\pi = 3$. ← False.

Remark: Context is important!

We take for granted the meaning of the symbols "1", "2", "+", "π", ...

In (ii) we implicitly assume that x is some real number (when writing such a statement in a mathematical argument one should make sure this is clear).

More examples:

(iv) 12 may be written as the sum of two primes.

(v) Every even integer greater than two may be written as the sum of two primes.

- $12 = 5 + 7$ so (iv) is true, in particular, (iv) is a proposition.
- Nobody knows for sure if (v) is true or false, but it must be one or the other. (Goldbach Conjecture)

What about the following?

a predicate → (vi) $n^2 - 2n > 0$. } Not propositions

(vii) π is a special number.
meaning?

(viii) π is an irrational number. ← True

(ix) $2+3$ } Not propositions.

(x) Math 109 is a great class.

Context is important here!

what does this mean?

(2) Predicates: A predicate is a sentence like (vi) that involves one or more free variables (like "n") which must be assigned a value before the truth or falsehood of the sentence can be decided.

Definition: A statement is a sentence that is either a proposition or a predicate.

1.2 Logical Connectives

Mathematical statements are often rather complicated, and are made up of various parts connected by "and", "or", ---- .

(1) The connective "or":

Example: For integers a, b, $ab = 0$ if $a = 0$ or $b = 0$.

The statement "$a=0$ or $b=0$" is true if $a=0$ and is true if $b=0$ (of course it is therefore also true if both $a=0$ and $b=0$).

Let P and Q be two statements.

Notation: $P \vee Q$ means "P or Q"

"disjunction"

Example: $a=0 \vee b=0$ means "$a=0$ or $b=0$"

Truth table for "or":

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Inclusive vs Exclusive: Note that "or" is ambiguous; "P or Q" could mean "at least one of the statements P and Q are true" ← inclusive or it could mean "exactly one of the statements P and Q is true".

"exclusive or" (excludes the case where both P and Q are true)

→ We will always use the "inclusive or" in math, given by the previous truth table.

$$\boxed{\vee \text{ means "inclusive or"}}$$

Examples:

- $a \leq b$ means $a < b$ or $a = b$.
- $a = \pm b$ means $a = b$ or $a = -b$.

Question: Are the following propositions true?

(i) $1 \leq 2$. (ii) $2 \leq 2$. (iii) $1 = \pm 1$.

True. True. True.

(2) The connective "and":

Let P, Q be two statements. The statement "P and Q" means "P holds and Q holds".

Notation: $P \wedge Q$ means "P and Q".

Truth table for "and"

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: $-1 < x < 1$ means $x > -1$ and $x < 1$.

(3) Negation ("not"):

Let P be a statement.

Notation: $\neg P$ means "not P" (i.e. "P does not hold")

Truth table for "not"

P	$\neg P$
T	F
F	T

Example:

(a) P : $x > y$.

$\neg P$: $x \le y$.

(b) P : $\pi = 1$.

$\neg P$: $\pi \neq 1$.

Question: Let P be the statement:

" For every integer a, if $a > -1$ then $a \geq 0$. "

Find $\neg P$. Is it true?

Solution:

$\neg P$: "There exists an integer a such that $a > -1$ and $a < 0$."

P is true, $\neg P$ is false.