Team Number:

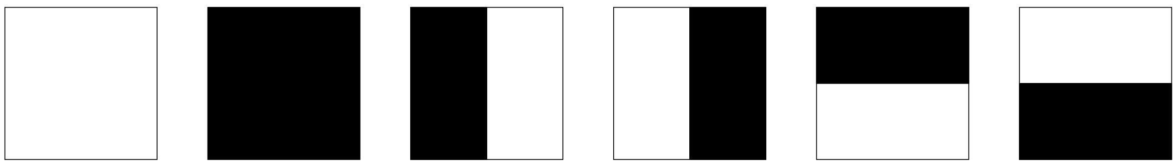
1. How many 4-digit numbers are divisible by 11, have a 5 in the hundreds place, and have a 7 in the ones place? (517 is not a 4-digit number.)

Solution: Any such number must have the form $A5B7$, with A and B digits between 0 and 9, and $A \neq 0$. For our number to be divisible by 11, we require $7 - B + 5 - A$ to be divisible by 11. That is, $A + B$ must equal 1 or 12. Note that this means A cannot be equal to 2. However, every other choice for A allows a unique B that works. Thus there are $\boxed{\text{eight}}$ such numbers:

1507	3597
4587	5577
6567	7557
8547	9537

Team Number:

2. A league of nations decides that commerce can be streamlined if all nations standardize the design of their flags. Under the new rules, all flags must be squares, consisting of two equally thick stripes (which can be both vertical or both horizontal), and each stripe must be one of ten standardized colors. Both stripes are allowed to be the same color, so for example there are six flags using the colors black and white:



How many distinct flags can be created?

Solution: There are $\dfrac{(10)(9)}{2} = 45$ ways to choose two colors, and each pair of colors provides four flags that actually use both colors. This accounts for 180 two-colored flags. Additionally, there are 10 one-colored flags, for a total of $180 + 10 = \boxed{190}$ flags.

Team Number:

3. A **Harshad number** is a number that's divisible by the sum of its digits. Some examples of Harshad numbers are 10 (divisible by $1 + 0$) and 27 (divisible by $2 + 7$). 2025 is Harshad, as were 2024, 2023, and 2022. What is the next Harshad year?

Solution: 2026 is not divisible by 10, 2027 is not divisible by 11, but $\boxed{2028}$ is divisible by 12.

All one-digit numbers are Harshad. 2025 concludes the third run of four consecutive Harshad years since the trivial ones $1, 2, \ldots, 10$: The previous such runs ended in 513 and 1017. The next sequence of four consecutive Harshad years will begin in 3030.

Team Number:

4. Find the last four digits of 2025^{2025}.

Solution: Observe that $2025 = 25 + 2000$, so

$$2025^n = 25^n + n(2000)(25)^{n-1} + \frac{n(n-1)}{2}(2000)^2(25)^{n-2} + \cdots + 2000^n.$$

Only the first two terms of this expansion have anything other than 0000 in the last four digits. Taking $n = 2025$ and ignoring all but the last four digits, we see that $(2025)(2000)(25)^{2024}$ ends with 0000 as well, so we are looking for the last four digits of 25^{2025}. We compute the last four digis of 25^n for small n, hoping to find a pattern:

n	End of 25^n
1	0025
2	0625
3	5625
4	0625
5	5625

So 25^n ends in 0625 if n is even and in 5625 if n is odd. Since 2025 is odd, the answer is $\boxed{5625}$.

Team Number:

5. A **Matryoshka square** is a square number from which another positive square number can obtained by deleting one digit. Two examples of Matryoshka squares are 16 (deleting the 6 leaves 1^2) and 2025 (deleting the 0 leaves 15^2).

   2025 is an **order two Matryoshka square** because 225 is also a Matryoshka square.

   There are 18 Matryoshka squares less than 2025, including four of order two and three of order 3. Score n points by finding an order n Matryoshka square other than 16, 225, or 2025.

Solution: We compute and organize the first 45 squares:

Square	Matryoshka Square	Order 2	Order 3	Square	Matryoshka Square
1	16	169	1369	400	
		196	1296	441	
			1936	484	
	81	841		529	
4	49			576	
	64			676	
9				729	
25	225	1225		784	
		2025		900	
	256			961	
	625			1024	
36	361			1089	
100	1600			1156	
121	1521			1681	
144	1444			1764	
289				1849	
324					