1. 2016 non-empty bags contain candy bars, and no two bags contain the same number of candy bars. What is smallest possible total number of candy bars?

Solution: The first bag must contain one bar, the second must contain two, and so on, until the last bag contains 2016 candy bars.

So the answer is

$$1 + 2 + 3 + \cdots + 2016 = \frac{(2016)(2017)}{2} = \boxed{2033136}$$

2. If $x + y = xy = 3$, what is $x^2 + y^2$?

Solution: We have

$$(x + y)^2 = x^2 + 2xy + y^2 = (x^2 + y^2) + 2(xy).$$

Plugging in the given values,

$$3^2 = (x^2 + y^2) + 2(3),$$

i.e., $x^2 + y^2 = \boxed{3}$.

3. The ten Big XII mascots agreed to share a block of hotel rooms during a tournament, and to split the costs equally. But the Longhorn decided to stay somewhere else at the last minute, and everybody else had to pay an extra \$22.40 to cover his share of the bill. What was the total cost of the block of rooms?

Solution: The Longhorn's share of the room block is being split nine ways, so he was supposed to pay $9(\$22.40) = \$201.60$.

Since the Longhorn was responsible for one tenth of the block, the total cost was $10(\$201.60) = \boxed{\$2016}$.

4. A robot starts at the origin in the Euclidean plane, and wishes to reach the point $(8, 4)$ by only taking unit-length steps directly up or to to the right. How many such paths are there that the robot could take from the origin to $(8, 4)$?

Solution: The robot is taking twelve steps, of which eight are to the right and four are up. Thus the number of possible paths is the same as the number of ways to choose four integers between one and twelve (corresponding to the four up steps). This number is the binomial coefficient $\binom{12}{4} = \frac{12!}{8!4!} = \boxed{495}$.

5. Say that a positive integer N is *nice* if it satisfies two properties:

   - All its digits are nonzero.
   - The integer M created by moving the rightmost digit of N to its far left satisfies $2M = 3N$.

     The smallest nice integer is $N = 285714$ (so $M = 428571$, and indeed $2M = 3N = 857142$).

   Find another nice integer.

Solution: Suppose N is nice, and write $N = 10A + B$, where B is its last digit. Then $M = 10^k B + A$, where k is the number of digits in A.

Since $2M = 3N$, we conclude that $(2 \times 10^k - 3)B = 28A$. Since $(2 \times 10^k - 3)$ is odd, we conclude that 4 divides B. Since none of the digits are zero, it follows that $B = 4$ or $B = 8$.

If $B = 4$, it follows that $7A = 1999 \ldots 9997$. Performing long division (and allowing the string of nines to terminate only where it would leave no remainder), we get $A = 28571$ or $A = 28571428571$ or $A = 28571428571428571$ or $\ldots$

If $B = 8$, it follows that $7A = 3999 \ldots 9994$. Performing long division (and terminating only where appropriate), we get $A = 57142$ or $A = 57142857142$ or $A = 57142857142$ or $\ldots$

Thus there are two nice integers with d digits, whenever d is divisible by six. The two six-digit nice integers are 285714 and $\boxed{571428}$. The two twelve-digit nice integers are $\boxed{285714285714}$ and $\boxed{571428571428}$. In general, all nice integers can be written by repeating either 285714 or 571428 any number of times.