

Hints for the 2025 OSU individual competition

1. The segment connecting the centers is a radius of both circles.
2. 365 days will have passed, and $365 = 52(7) + 1$.
3. $3^{-2x} = \frac{1}{(3^x)^2}$.
4. $\angle EDH = 180^\circ - \frac{360^\circ}{5}$. $\angle EDC = 45^\circ$.
5. There are 31 possible days of the month, but more than 31 students.
6. The sides of a triangle satisfy $a + b > c$. No two odd primes differ by less than 2, so all three sides must be at least 3.
7. If she had x_5 eggs before the last customer, she sold $\frac{1}{2}x_5 + \frac{1}{2}$. Since she had no eggs left after this transaction, we know $x_5 - (\frac{1}{2}x_5 + \frac{1}{2}) = 0$. Solving this for x_5 allows us to write a similar equation for x_4 , the number of eggs before the fourth customer, and so on.
8. $(2 + 3i)^2 = 2^2 + 12i + (3i)^2 = 4 + 12i - 9 = -5 + 12i$. Then $(2 + 3i)^4 = (-5 + 12i)^2$.
9. Before the slices, the cheeseblock has $\ell + h + w = 29$. After, it has $\ell + h + w = 24$. The volume is maximized by making ℓ, h, w as close to equal as possible. But $\ell \leq 6$, so we want $w = h = 9$.
10. Imagine that the second hand is a half-lap ahead of the minute hand at the beginning of class. During class, the second hand makes exactly 50 additional laps, while the minute hand makes almost one. Thus the second hand ends class ahead by $\frac{1}{2} + 50 - (\text{almost one})$ laps. The lead is thus more than 49 but less than 50.
11. We have $y = x + d$ and $z = y + d$. The common ratio is $r = \frac{y+d}{y} = \frac{x+d}{x}$. Thus $r - 1 = \frac{d}{y} = \frac{d}{x}$. This requires $d = 0$ or $x = y$; either way $r = 1$.
12. Draw lines from the center of the circle to the three vertices and the three points of tangency. We get six 30-60-90 triangles.
13. The left-hand side is almost, but not quite, equal to $(x + 1)^4$. If we set $y = (x + 1)$, can we find the two real solutions for y ?
14. Draw a segment through H parallel to AB . The resulting right triangle allows us to compute AB . Meanwhile, $AC = 9 + 12$.
15. We know $\ln(2025) = 4 \ln 3 + 2 \ln 5$. Thus $a = 4$, $b = 0$, $c = 2$, as any other solution would give us a different prime factorization of 2025.
16. Notice that P has area 12. Our line $y = mx$ must pass outside P at a point $(x, 5)$ on the segment between $(1, 5)$ and $(2, 5)$. The region to the right is a trapezoid with bases 2 and $2 - x$, and height 5; its area must equal 6.

17. Set $x = \sqrt[3]{2} + \sqrt[3]{4}$. Then $x^2 = \sqrt[3]{4} + 2\sqrt[3]{2} + 4$, and $x^3 = 6 + 6\sqrt[3]{2} + 6\sqrt[3]{4}$.
18. Writing $n = 2^r s$, we get $S(n) = (1 + 2 + \cdots + 2^r)(S(s))$, since expanding this product will add every divisor of s multiplied by every power of 2 exactly once. Thus $S(s)$ must be odd, which requires s to have an odd number of factors; that is, s must be a perfect square. So we are looking for a year N which is a perfect square times a power of 2. The next such year is $2048 = 2^{11} \times 1$.
19. Placing the origin at the base of the wall, the two endpoints of the pole must be at $(a, 0)$ and $(0, b)$ for some a and b ; by the Pythagorean theorem, we have $a^2 + b^2 = \ell^2$. If the midpoint is at (x, y) , then $x = \frac{a}{2}$ and $y = \frac{b}{2}$ must satisfy $x^2 + y^2 = \frac{1}{4}\ell^2$.
20. To pass through point C he must first reach point $(0, 0)$ at the northeast corner of building A (which happens with probability 1, then get to point $(1, 2)$ to the West of C (which happens whenever the first three coin tosses come up North, North, and East in some order), then turn East (which, conditioned on being at $(1, 2)$, happens with probability $\frac{1}{2}$). The fact that there are only 20 paths from A to B is irrelevant, since paths that reach the North or East boundary of the picture early have a higher probability than ones that don't.
21. We have $BG = 20$ and $BH = DI = 25$. Now apply the power of a point theorem to find BE .
22. Since $1^3 + 2^3 + \cdots + 9^3 = 2025$, the large cubes are $1 \times 1 \times 1$ through $9 \times 9 \times 9$. The invisible cubes are on the interiors of the big cubes, which have dimensions $(n-2) \times (n-2) \times (n-2)$ inside a cube of size $n \times n \times n$. Thus there are $1^3 + 2^3 + \cdots + 7^3 = 784$ invisible small cubes. To guarantee a visible black cube, we need one more.
23. Drop a perpendicular from A to EF . It meets EF at the point of tangency G with the left circle, and has length r . Now triangles ECF and EGA are similar.
24. Observe that $f(t) - t = 1$ whenever $t = \frac{x}{1-x}$. Since f is continuous we have $f(x) = 1 + x$. Thus the answer is $1 + \sin \theta$. Plugging in $\theta = 0$ and $\theta = \frac{\pi}{2}$ will eliminate four of the answer choices. To verify that the survivor is correct, rewrite with sines and cosines only, simplify as much as possible, and remember that $\cos^2 \theta = 1 - \sin^2 \theta$ and $1 - a^2 = (1-a)(1+a)$.
25. Rotate the picture ninety degrees clockwise about A , creating new points A' , B' , $\dots$, F' . Draw EF' and find a pair of congruent triangles and an opportunity to use the Pythagorean theorem.