Team Number:

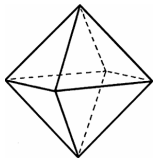
1. Find all the four-digit palindromes that are divisible by 18.

   [A palindrome is a number that reads the same from left-to-right as from right-to-left. For example, 1221 is a four-digit palindrome, but it isn't divisible by 18.]

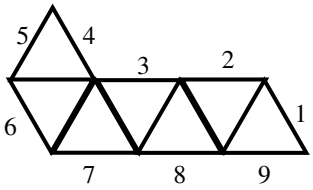
   [A four-digit number is a number between 1000 and 9999 (inclusive). For example, the number 0000 does not count as a four-digit palindrome.]

Team Number: ____

2. An **octahedron** is a regular polyhedron with eight triangular sides, as in the picture.



A **net** of an octahedron is an arrangement of eight equilateral triangles in the plane that can be folded to form an octahedron. The following diagram of seven equilateral triangles can be extended to a net by adding one more triangle along which edge(s)?



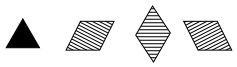
Team Number: [ ]

3. There are fourteen ordered pairs (a, b) with the property that the polynomial $x^3 - ax^2 + bx - 2024$ has three distinct positive integer zeros. What is the smallest possible value of a?

[ ]

Team Number:

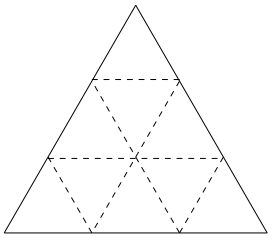
4. A triangular region can be subdivided into smaller triangles and then tiled with translates of the three diamond tiles and the upward-pointing triangle:



There are exactly three ways to tile an equilateral triangle of side-length two:

(Rotations aren't allowed, so the downward-pointing triangle isn't a tile, which means that arrangements like the picture on the right don't count as tilings.)

There are exactly eighteen ways to tile the equilateral triangle of side-length three:



If we were to build all eighteen of these tilings simultaneously, how many copies of the upward-pointing triangle would we need?

(If the question were about the triangle with side 2, the answer would be six, since we want to display all the tilings at once.)

Team Number:

5. At halftime of the homecoming game, Pistol Pete stations 101 giant foam cowboy hats on the football field, one on each yard line (and the two goal lines). One lucky fan is chosen from the student section, and told to pick a hat. Before the fan picks the hat, Pete explains that each hat covers some hundred-dollar bills: The hat on the n-yard line contains a number of bills equal to the number of hats containing exactly n hundred-dollar bills.

   (The OSU goal line is considered the 0-yard line, and the opposing goal line is considered the 100-yard line. Similarly, the yard lines between the 50-yard line and the 100-yard line count upward, so for example the opposing 30-yard line is considered the 70-yard line, and so on.)

   Which hat should the fan pick? If she chooses her hat wisely, how much money does she get?