1. 2023 is to be factored as a product of three positive integers, $2023 = abc$ with $a < b < c$. How many possible values could $b$ take?

   (a) 0
   (b) 6
   (c) 4
   (d) 7
   (e) 2

2. Find the difference between $O$, the sum of the positive odd numbers less than or equal to 2023, and $E$, the sum of the positive even numbers less than 2023.

   (a) 0
   (b) 1
   (c) 2
   (d) 1011
   (e) 1012

3. A ball falls into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area, $A$, of the circle in terms of the time, $t$ (in seconds), that has passed since the ball hit the lake.

   (a) $A(t) = 25\pi t$ cm$^2$
   (b) $A(t) = \pi t^2$ cm$^2$
   (c) $A(t) = 25\pi t^2$ cm$^2$
   (d) $A(t) = 5\pi t^2$ cm$^2$
   (e) $A(t) = 10\pi t$ cm$^2$

4. A **Pythagorean triple** is a list of three positive integers $0 < a < b < c$ with the property that $a^2 + b^2 = c^2$. (Some examples of Pythagorean triples are $(3, 4, 5)$, $(6, 8, 10)$, and $(8, 15, 17)$.) Which of the following statements must be true for a Pythagorean triple $(a, b, c)$?

   I: $a + b > c$.

   II: $a + b < c$.

   III: $a^3 + b^3 > c^3$.

   IV: $a^3 + b^3 < c^3$.

   (a) I only.

   (b) II and III.

   (c) II and IV.

   (d) None of these.

   (e) I and IV.

5. If five cowboys can mow six lawns in seven hours and seven sooners can mow five lawns in six hours, how many lawns can a team of two cowboys and three sooners mow in forty hours?

   (a) 5

   (b) 17

   (c) 25

   (d) 28

   (e) 30

6. What is the last digit of $2023^{2023}$?

   (a) 1

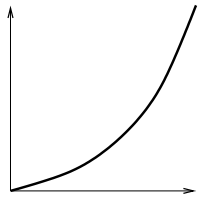
   (b) 3

   (c) 5

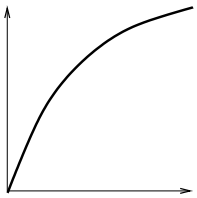
   (d) 7

   (e) 9

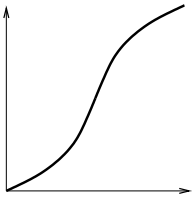
7. A fountain fills a spherical bottle with water at a constant rate of 1 gallon per minute. Which of the following graphs could represent the height of the water in the bottle as a function of the volume of water in the bottle?



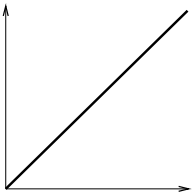
Graph I



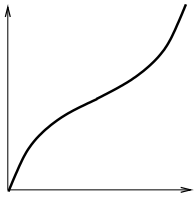
Graph II



Graph III



Graph IV



Graph V

(a) Graph I.

(b) Graph II.

(c) Graph III.

(d) Graph IV.

(e) Graph V.

8. Which of the numbers below is the largest?

(a) $\dfrac{10^{2020}-1}{10^{2021}-1}$

(b) $\dfrac{10^{2021}-1}{10^{2022}+1}$

(c) $\dfrac{10^{2022}+1}{10^{2023}+1}$

(d) $\dfrac{10^{2023}+1}{10^{2024}-1}$

(e) $\dfrac{10^{2024}-1}{10^{2025}-1}$

9. Find the center of the circle $x^2 + y^2 + 7x - 17y = 2023$.

   (a) $\left(-\frac{7}{2}, -\frac{17}{2}\right)$

   (b) $\left(-\frac{17}{2}, \frac{7}{2}\right)$

   (c) $\left(-\frac{17}{2}, -\frac{7}{2}\right)$

   (d) $\left(\frac{17}{2}, \frac{7}{2}\right)$

   (e) $\left(-\frac{7}{2}, \frac{17}{2}\right)$

10. Find the remainder when $x^{10} - 10$ is divided by $x^2 - 2$.

    (a) 0

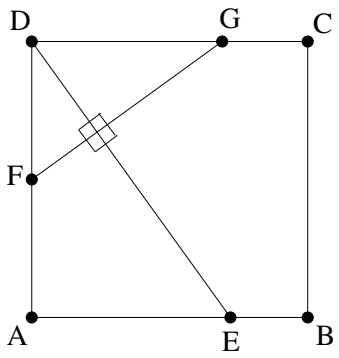
    (b) 5

    (c) 22

    (d) 50

    (e) 96

11. In square $ABCD$, $AE = 9$ and $EB = 3$. If $DF = FA$ and $FG$ is perpendicular to $DE$, find $CG$.




    (a) 4

    (b) 5

    (c) 6

    (d) 7

    (e) 8

12. A student shows up to school and claims that the dog ate her math homework. As it turns out, this is true, and the page is half chewed up. The problem seems to say

    Find the roots of the polynomial

    $$x^4 - 5x^3 - 8x^2 + (\text{remainder is illegible})$$

    The student's work is also torn up, but it looks as if her first three solutions are $x = -2$, $x = 0$, and $x = 1$. Assuming that her work was correct, what is the missing solution?

    (a) -5
    (b) -3
    (c) 3
    (d) 6
    (e) 9

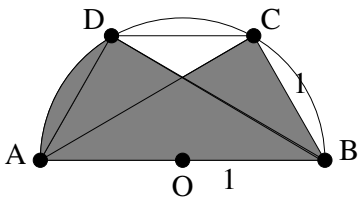
13. If $y = \dfrac{2x}{2x+3}$, what is $x$ as a function of $y$?

    (a) $\dfrac{3y}{2y+2}$
    (b) $-\dfrac{2y-2}{3y}$
    (c) $-\dfrac{3y}{2y-2}$
    (d) $-\dfrac{3y-2}{2y}$
    (e) $\dfrac{3y}{2y-2}$

14. Charlotte's piggy bank contains 100 coins, all of which are nickels, dimes, and quarters. If Charlotte's quarters were nickels and her nickels were quarters, the total value of her piggy bank would increase by \$2.40. If Charlotte's quarters were dimes and her dimes were quarters, the total value would increase by \$1.95. How much money does Charlotte have?

    (a) \$11.85
    (b) \$11.90
    (c) \$11.95
    (d) \$12.00
    (e) \$12.05

15. In the figure, $AB$ is a diameter of semicircle $O$, which has radius 1. If $BC = 1$ and $CD$ is parallel to $AB$, find the area of the shaded region.




(a) $\dfrac{\pi + 3\sqrt{3}}{6}$

(b) $\dfrac{2\pi + 5\sqrt{3}}{12}$

(c) $\dfrac{\pi + 2\sqrt{3}}{6}$

(d) $\dfrac{2\pi + 3\sqrt{3}}{12}$

(e) $\dfrac{\pi + \sqrt{3}}{6}$

16. Recall that, for a real number $t$, the expression $\lfloor t \rfloor$ represents the "floor" of $t$, that is, the greatest integer less than or equal to $t$. For example, $\lfloor 2.9 \rfloor = 2$ and $\lfloor 3 \rfloor = 3$.

    For how many positive integers $x \leq 2023$ is $\lfloor \log x \rfloor$ even? (Recall that "log" stands for the logarithm base ten.)

(a) 909

(b) 1010

(c) 1111

(d) 1212

(e) 1313

17. An integer $N$ is chosen at random between 1 and 20 (inclusive). What is the probability that $N+1$ is a divisor of $N^2+4N+23$?

(a) $\frac{1}{4}$

(b) $\frac{3}{10}$

(c) $\frac{7}{20}$

(d) $\frac{2}{5}$

(e) $\frac{9}{20}$

18. Define a sequence by the rule $x_1 = \frac{5}{4}$ and $x_{n+1} = \frac{16x_n^2 - 21}{16}$ for all $n$. Compute $x_{2023}$.

(a) $\frac{1}{4}$

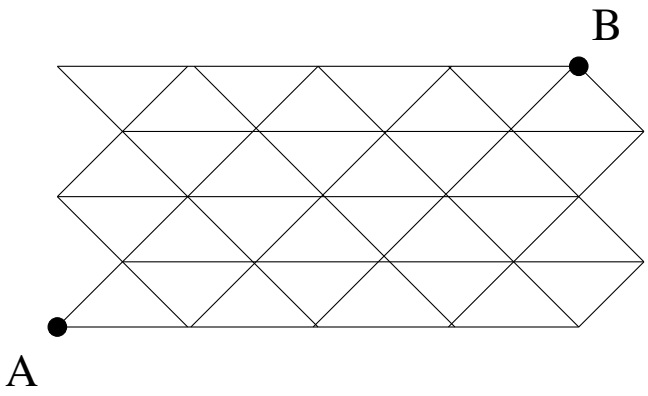
(b) $-\frac{1}{4}$

(c) $-\frac{5}{4}$

(d) $-\frac{21}{16}$

(e) $\frac{5}{4}$

19. A spider is at point $A$ of its web (shown below) when it captures a fly at point $B$. It then travels to point $B$ along the strands of its web. Assuming the spider only moves from left to right along the horizontal strands, and only upwards along the diagonal strands (with either slope), how many distinct paths could it take?




(a) 15

(b) 132

(c) 210

(d) 814

(e) 1289

20. A **False Fibonacci sequence** is a sequence of positive integers $\{f_1, f_2, f_3, \dots\}$ in which $f_{n+2} = f_{n+1} + f_n$ for all $n$ (but $f_1$ and $f_2$ are not necessarily equal to 1). If $f_1, f_2, \dots$ is a false Fibonacci sequence and $f_6 = 20$, find $f_7$.

(a) 31

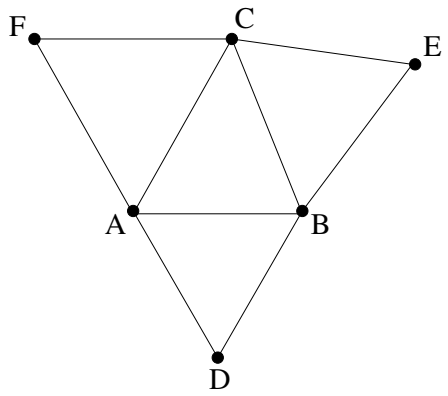
(b) 32

(c) 33

(d) 34

(e) 35

21. In the figure, triangles $ABD$, $BCE$, and $CAF$ are equilateral, and points $D$, $A$, and $F$ are collinear. If triangle $ABD$ has area 36, triangle $BCE$ has area 43, and triangle $CAF$ has area 49, find the area of triangle $ABC$.




(a) 42

(b) 43

(c) 44

(d) 45

(e) 46

22. A desk contains pages which are labeled with positive integers. A robot tidies the desk according to the following instructions:

- If there are papers on the desk, choose one at random and read its label. Then:
  - If the chosen paper was labeled with the number $N$, replace it on the desk with $N-1$ new pages, labeled with the numbers from 1 to $N-1$.
  - Recycle the chosen paper.
- If there are no papers on the desk, move on to the next office.

When the robot arrives, the desk contains a single page, labeled "2023". How many pages does the robot recycle before moving on?

(a) 2022!

(b) 2023

(c) 2,047,276

(d) 4,092,529

(e) $2^{2022}$

23. Find the sum of the squares of the roots of the polynomial $x^3 + 6x^2 + 8x + 2$.

(a) 12

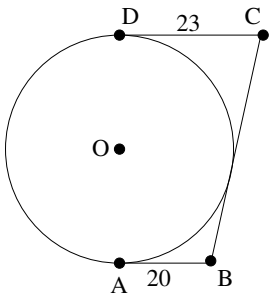
(b) 14

(c) 16

(d) 18

(e) 20

24. In the figure, $AB$, $BC$, and $CD$ are all tangent to circle $O$, and $AB$ is parallel to $CD$. If $AB = 20$ and $CD = 23$, find the area of circle $O$.




(a) $400\pi$

(b) $415\pi$

(c) $430\pi$

(d) $445\pi$

(e) $460\pi$

25. Simplify $\dfrac{\cos 23^\circ + \sin 23^\circ}{\cos 23^\circ - \sin 23^\circ}$.

(a) $\cot 22^\circ$

(b) $\csc 23^\circ$

(c) $\sec 24^\circ$

(d) $-1$

(e) $\tan 46^\circ$