1. How many ordered pairs of nonnegative integers satisfy the equation  $x^2 - y^2 = 2^{22}$ ? [One solution is  $(x, y) = (2^{11}, 0)$ ].



2. Find the area of the pentagon below.





3. Find 3 consecutive positive integers such that the sum of the square of the smallest integer and the square of half of the middle integer equals the square of the largest integer.

4. Say a 10 digit number  $A = a_0 a_1 \cdots a_9$  is **self-referential** if each  $a_i$  is the number of digits in A equal to *i*. Find a self-referential number.

5. A number is called **very prime** if it is prime and its base-ten representation is prime when viewed as a base-eight expression.

Some examples of very prime numbers are 2, 3, 5, 7, 13, and 2017. (13 is very prime because  $13_8 = 1 \times 8 + 3 \times 1 = 11_{10}$  is prime, and 2017 is very prime because  $2017_8 = 2 \times 8^3 + 0 \times 8^2 + 1 \times 8 + 7 \times 1 = 1033_{10}$  is prime. On the other hand, 11 is not very prime because  $11_8 = 1 \times 8 + 1 = 9_{10}$  is composite. Numbers like 19 and 83 are not very prime, the base-eight expressions  $19_8$  and  $83_8$  don't make sense.)

Find a very prime three-digit number.