1. The polynomial equation $20x+16 = (2x+0)(1x+6)$ has two roots. Find their product.

   A. -8
   B. -16
   C. -20
   D. -32
   E. -48

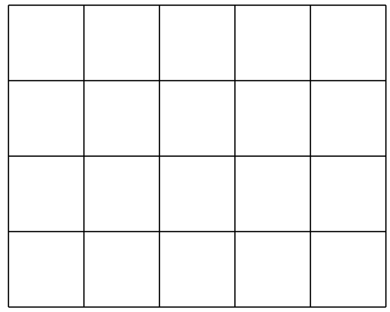
2. A college student spends \$12 to travel to a casino, where he loses half his money. He then spends \$26 to travel to a second casino, where he loses two-thirds of his remaining money. Next, he spends \$48 to travel to a third casino, where he loses three-quarters of his remaining money. Finally, he spends his last \$37 to return home. How much money did he start with?

   A. \$123
   B. \$888
   C. \$1240
   D. \$2016
   E. \$2912

3. Two fair six-sided dice are rolled. What is the probability that the sum of the results is equal to six?

   A. $\frac{1}{36}$
   B. $\frac{1}{12}$
   C. $\frac{5}{36}$
   D. $\frac{7}{36}$
   E. $\frac{1}{4}$

4. How many rectangles have all four of their sides among the lines in the grid below? (Assume the grid is what it looks like: six parallel vertical lines and five parallel horizontal lines.)



A. 50

B. 110

C. 120

D. 150

E. 400

5. How many circles of radius 2 are tangent to both $C_1 : (x-1)^2 + y^2 = 1$ and $C_2 : (x+1)^2 + y^2 = 1$?

A. 1

B. 2

C. 3

D. 4

E. 5

6. How many of the numbers 1, 2, 3, ..., 2016 are divisible by either 20 or 16?

A. 176

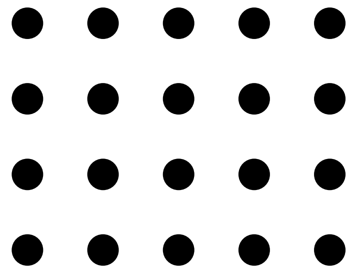
B. 201

C. 226

D. 251

E. 276

7. How many squares have all four of their vertices among the lattice points in the grid below? (Assume the grid is what it looks like: 20 points with x-coordinates chosen from $\{0, 1, 2, 3, 4\}$ and y-coordinates chosen from $\{0, 1, 2, 3\}$.)



A. 20

B. 24

C. 28

D. 30

E. 33

8. A football fan drives 80 miles from Norman to Stillwater to watch a game, and can only average 25 miles per hour in the heavy traffic. Later, he leaves before the end of the game and is able to average 75 miles per hour while driving home. What is his average speed for the whole trip?

A. 60 miles per hour

B. 40 miles per hour

C. 50 miles per hour

D. 25 miles per hour

E. 37.5 miles per hour

9. A census of OSU dorm rooms has found the fraction of rooms occupied by various numbers of students:

Number of students in room	Percentage of rooms
1	15%
2	50%
3	30%
4	5%

What fraction of students live in a double room (i.e., a room occupied by exactly two students)?

A. $\frac{3}{8}$

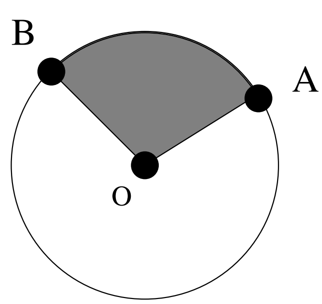
B. $\frac{4}{9}$

C. $\frac{1}{2}$

D. $\frac{5}{9}$

E. $\frac{5}{8}$

10. The circle $x^2 + y^2 = 4$ has center $O = (0, 0)$. If $A = (\sqrt{3}, 1)$ and $B = (-\sqrt{2}, \sqrt{2})$, find the area of sector AOB.



A. $4\pi - 4\sqrt{5}$

B. $\dfrac{\sqrt{6} + \sqrt{2}}{2}$

C. $\dfrac{7\pi}{6}$

D. $\dfrac{2\pi - 1}{2}$

E. $\dfrac{\pi + 2\sqrt{3}}{2}$

11. A dormitory at OSU houses 200 students, of whom 32 major in math, 18 major in English. If seven of these students double-major in both math and English. How many of this dormitory's residents major in neither math nor English?

   A. 129
   B. 136
   C. 143
   D. 150
   E. 157

12. Two sides of a right triangle have length 20 and 16. If the third side also has integer length, find the altitude to the side of length 20.

   A. $\frac{48}{5}$
   B. 10
   C. $\frac{52}{5}$
   D. $\frac{54}{5}$
   E. $\frac{56}{5}$

13. Guildenstern is listening to his iPod on 'shuffle.' His music library consists of 10 bands, each with 10 songs. Assuming the 'shuffle' function is random and fair, what is the probability that the first 4 songs Guildenstern listens to are all by different bands? (In 'shuffle' mode, the iPod plays each song exactly once, in random order.)

   A. Less than $\frac{1}{4}$
   B. Between $\frac{1}{4}$ and $\frac{1}{3}$
   C. Between $\frac{1}{3}$ and $\frac{1}{2}$
   D. Between $\frac{1}{2}$ and $\frac{2}{3}$
   E. Greater than $\frac{2}{3}$

14. Compute $\dfrac{(2+i)^3(2-i)^3}{i^{11}}$.

   A. $27i$
   B. $125i$
   C. $-27i$
   D. $-125i$
   E. $-65i$

15. A rectangular prism has faces with area 24, 32, and 48. Find its volume.

   A. 108

   B. 162

   C. 192

   D. 288

   E. 384

16. What pairs of real numbers (a, b) satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$?

   A. All pairs (a, b) with $a \neq 0$, $b \neq 0$, and $a \neq -b$.

   B. Pairs of the form $(a, \frac{1}{a})$ with $a \neq 0$ only.

   C. None.

   D. Pairs of the form $(a, \frac{a^2-1}{a})$ with $a \neq 0, \pm 1$ only.

   E. Pairs of the form (a, a) with $a \neq 0$ only.

17. A triangle has sides of length 13, 20, and 21. Find its area.

   A. 117

   B. 120

   C. 121

   D. 125

   E. 126

18. Let A and B be such that $\dfrac{20x + 16}{(x-5)(x-1)} = \dfrac{A}{x-5} + \dfrac{B}{x-1}$. Find $A - B$.

   A. 35

   B. 36

   C. 37

   D. 38

   E. 39

19. One circle drawn on the surface of a sphere divides it into two regions. Two circles divide the sphere into four regions (if they cross each other) and three regions (if they don't). What is the largest number of regions into which the sphere can be divided by four circles?

A. 11

B. 12

C. 13

D. 14

E. 15

20. What is the sum of the positive factors of 2016?

A. 6552

B. 6553

C. 6554

D. 6555

E. 6556

21. A solid wooden box has dimensions $2 \times 3 \times 4$. An ant stands at one corner of the box, and food is placed on the opposite corner. Assuming the ant can move equally quickly along all faces of the box, but cannot pass through its interior, what is the length of its shortest path to the food?

A. $\sqrt{29}$

B. $\sqrt{33}$

C. $\sqrt{37}$

D. $\sqrt{41}$

E. $3\sqrt{5}$

22. The prime factorization of a positive integer n is an expression $n = p_1^{r_1} p_2^{r_2} \dots p_k^{r_k}$ where $p_1 < p_2 < \cdots < p_k$ are prime numbers and the exponents $r_1, \dots, r_k$ are positive integers. Every positive integer except 1 has exactly one prime factorization.

Define a function f, with domain the positive integers, as follows. Set $f(1) = 1$ and, for $n > 1$, set $f(n) = r_1^{p_1} r_2^{p_2} \dots r_k^{p_k}$, where $n = p_1^{r_1} p_2^{r_2} p_k^{r_k}$ is the prime factorization of n.

Among the statements

I. f is one-to-one,

II. Every positive integer occurs as $f(n)$ for some n,

III. If $m \leq n$, then $f(m) \leq f(n)$,

IV. If $\gcd(m, n) = 1$, then $f(mn) = f(m)f(n)$,

which are correct?

A. **I**, **II**, and **III**.

B. **II** and **III**.

C. **II**, **III**, and **IV**.

D. **IV** only.

E. **II** only.

23. A sequence a_n starts with two arbitrary real numbers a_1 and a_2 and continues on according to the formula $a_{n+2} = \max\{a_{n+1} - 1, a_n + 1\}$. If $a_{100} = 100$, what is the largest possible value of a_1?

A. 52

B. 53

C. 54

D. 55

E. 56

24. Let ABC be a triangle with area 1. Let M, N, P be the midpoints of BC, AC, and AB, respectively. Assume that AM intersects NP and NB in Q and G, respectively. What is the area of NQG?

A. $\frac{1}{8}$

B. $\frac{1}{6}$

C. $\frac{1}{24}$

D. $\frac{1}{12}$

E. $\frac{1}{16}$

25. An integer function $f : \mathbb{Z} \to \mathbb{Z}$ is called *super* if, for all integers x and y,

$$f(x+y) + f(xy) = f(x)f(y) + 1.$$

How many super functions exist?

A. None.

B. One.

C. Two.

D. Finitely many, but more than two.

E. Infinitely many.