

Team Name:

1. Remember that a *palindrome* is a number (or word) that reads the same backwards and forwards. For example, 353 and 2112 are palindromes.

Observe that the base 2 representation of 2015 is a palindrome. Find the next year with a palindromic base 2 representation. (Express your answer in base ten.)

(The base ten representation of 2015 is 2015, because 2, 0, 1, and 5 are the coefficients in the expression

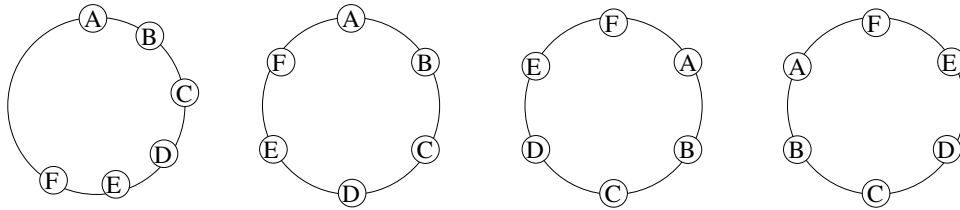
$$2015 = 2(10^3) + 0(10^2) + 1(10^1) + 5(10^0).$$

The base two representation is the sequence of coefficients when we replace those powers of ten with powers of two. For example, the base two representation of  $17 = 2^4 + 2^0$  is 10001, and the base two representation of 2015 is 11111011111.)

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2. A closed necklace is to be made from six different jewels. How many different types of necklaces can be made?

(We say two necklaces are the same type if one can be obtained from the other by sliding the jewels along the chain (but not across one another), by rotating the entire necklace, or by turning it over. Thus, the four necklaces below are all of the same type.)



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3. For any two people, we assume that there are only two options: Either the two people are strangers to each other or they are friends. What is the smallest number of people you need to invite to a party so that you are guaranteed that either (i) there are three people there who are all strangers to one another or (ii) there are three people there who are all friends?

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4. A set is *felicitous* if it satisfies two properties:

- Its elements are all integers between 1 and 2015, inclusive.
- The product of any two of its elements is a perfect square.

What is the size of the largest possible felicitous set?

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5. A *primitive Pythagorean quadruple* is an ordered quadruple of positive integers  $(a, b, c, d)$  satisfying  $a \leq b \leq c \leq d$ ,  $a^2 + b^2 + c^2 = d^2$  and  $\gcd(a, b, c, d) = 1$ . One example of a primitive Pythagorean quadruple is  $(1, 2, 2, 3)$ . Find three others.
