1. What is the value of the product

$$2^{1/2} \cdot 2^{1/4} \cdot \ldots \cdot 2^{1/2^n} \cdot \ldots ?$$

   A. ∞
   B. 1
   C. 2
   D. 4
   E. e

2. If a polygon is obtained by intersecting a triangle with a square, then what is the maximum possible number of its sides?

   A. 3
   B. 4
   C. 5
   D. 6
   E. 7

3. A pair of fair six-dided dice is rolled. What is the probability of getting a sum of 2?

   A. 1/6
   B. 1/3
   C. 1/36
   D. 5/16
   E. 1/4

4. What is the solution set to $x^2 - 20x < 25$?

   A. $\left(10 - 5\sqrt{5}, 10 + 5\sqrt{5}\right)$
   B. $\left[10 - 5\sqrt{5}, 10 + 5\sqrt{5}\,\right]$
   C. $\left(-\infty, 10 - 5\sqrt{5}\right) \cup \left(10 + 5\sqrt{5}, \infty\right)$
   D. $\left\{10 - 5\sqrt{5}, 10 + 5\sqrt{5}\right\}$
   E. $\left(-\infty, 10 - 5\sqrt{5}\,\right] \cup \left[10 + 5\sqrt{5}, \infty\right)$

5. Suppose that, in some base b, $6_b \times 9_b = 42_b$. Then, still working in base b, what is $20_b \times 10_b$?

   A. 13_b
   B. 125_b
   C. 200_b
   D. 225_b
   E. 256_b

6. How many integers between 1 and 2010 are divisible by all of $1, 2, 3, 4, 5, 6, 7$, and 8?

   A. 0
   B. 1
   C. 2
   D. 4
   E. 8

7. Let $f(x) = x^4 - 40x^3 + 501x^2 - 40x + 500$. Given that $x = 20 + 10i$ is a solution to $f(x) = 0$, how many distinct real roots does f have?

   A. 0
   B. 1
   C. 2
   D. 3
   E. 4

8. Find the remainder when $2x^3 + x$ is divided by $x + 1$.

   A. 0
   B. $3x$
   C. $x - 3$
   D. -3
   E. 1

9. The diagonals of a convex quadrilateral are perpendicular and have lengths 4 and 8. The area of the quadrilateral is

   A. 16
   B. 20
   C. 32
   D. 60
   E. Cannot be determined from the given information.

10. Given a triangle ABC such that $AB = \sqrt{2}, BC = 2$ and $\angle C = 30°$, the measure of the angle B is

A. $15°$
B. $45°$
C. $45°$ or $135°$
D. $15°$ or $105°$
E. Cannot be determined from the given information.

11. In a shipment of 100 televisions, 6 are defective. If a person buys two televisions from that shipment, what is the probability that both are defective?

A. 3/100
B. 1/200
C. 9/2500
D. 1/330
E. 3/200

12. The set of solutions to the inequality

$$\frac{1}{\log_2 x} - \frac{1}{-1 + \log_2 x} < 1$$

is given by

A. $(0, \infty)$
B. $(0, 1) \cup (4, \infty)$
C. $(0, 2) \cup (3, \infty)$
D. $(-\infty, 1) \cup (2, \infty)$
E. $(0, 1) \cup (2, \infty)$

13. The expression

$$\frac{\log_a x}{\log_{ab} x}$$

can be written as

A. $\log_a x - \log_b x$
B. $\log_a x + \log_b a$
C. $1 + \log_a b$
D. $1 + \log_a x$
E. $1 - \log_b x$

14. For which value of a do the graphs of $y = x^2 + ax + 2$ and $y = 2x^2 - ax + 6$ have exactly one point of intersection?

A. $\sqrt{2}$

B. 2

C. 3

D. π

E. 4

15. Find an equation of the circle inscribed into the rhombus with vertices $(1, 3)$, $(5, 6)$, $(9, 3)$ and $(5, 0)$.

A. $(x - 5)^2 + (y - 3)^2 = 4$

B. $(x - 3)^2 + (y - 5)^2 = 4$

C. $(x - 5)^2 + (y - 3)^2 = 144/25$

D. $(x - 5)^2 + (y - 3)^2 = 64/9$

E. None of the above.

16. Alice, Bob, Carl, and Dorothy must be seated on one side of a long table, but someone needs to sit between Alice and Bob. In how many distinguishable ways can these four people be seated at the table?

A. 10

B. 12

C. 16

D. 20

E. 22

17. The points A, B, C are on a circle of radius 5, and the segment AB has length 8. The maximum possible area of the triangle ABC is

A. 10

B. 20

C. 24

D. 32

E. 36

18. If a cowboy breaks as many horses in a day as a sooner does in a week, and three sooners working together can break four horses in five hours, how long will it take a team of twenty sooners and ten cowboys to break a herd of 120 horses?

A. 1 hour

B. 2 hours

C. 3 hours

D. 4 hours

E. 5 hours

19. The area of a triangle is 10, and its perimeter is 20. The radius of its inscribed circle is

A. 1

B. $\frac{5}{4}$

C. 8

D. 10

E. Cannot be determined from the given information.

20. Simplify: $\sqrt{3+\sqrt{3+\sqrt{3+\cdots}}} =$

A. $\frac{\sqrt{5}-1}{2}$

B. $\frac{\sqrt{13}-1}{2}$

C. $\frac{1+\sqrt{13}}{2}$

D. $\frac{7+\sqrt{13}}{2}$

E. $\frac{7-\sqrt{13}}{2}$

Part II. Complete solution problems

1. Diagonals AC and BD of a trapezoid $ABCD$ intersect at point O. Assuming that AD is parallel to BC, prove that the areas of triangles ABO and CDO are equal.

2. Show that $a^4 + b^4 \geq 1/8$ for all real numbers a and b satisfying $a + b = 1$.

3. Let $A = (c, 0)$ and $B = (-c, 0)$ be distinct points in the plane, and let k be a positive number different from 1. Show that all points P such that $|\overline{AP}|/|\overline{BP}| = k$ lie on a circle with center on the line through A and B. What is the location of points P when $k = 1$?

4. What are the last 2015 digits of 2010^{2010}? (For example, the last three digits of 2010 are 010.)

5. Consider an 8×8 chessboard with two squares in diagonally opposite corners removed. Is it possible to tile this board with 2×1 dominoes?