1. Which of the numbers below is a multiple of 2024?

   (a) 1890305

   (b) 1890306

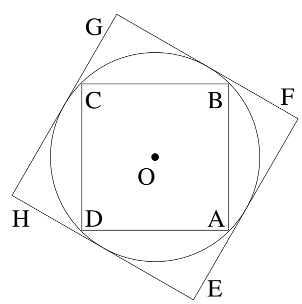
   (c) 1890316

   (d) 1890416

   (e) 1891416

   Answer: D.

2. Square $ABCD$ is inscribed in circle O, which is inscribed in square $EFGH$. If the area of $ABCD$ is 24, find the area of $EFGH$.



   (a) 24

   (b) $24\sqrt{2}$

   (c) 36

   (d) 12π

   (e) 48

   Answer: E.

3. The Big XII conference contains sixteen football teams. They want a fair way to determine a champion, so that no team can object that they were hurt by an unbalanced schedule or by another team's home-field advantage. After extensive discussions, the teams agree to play a full double round-robin schedule, so that each team plays two games against each other team (one on each home field).

   How many total games will be played?

   (a) 32
   (b) 120
   (c) 225
   (d) 240
   (e) 256

   Answer: D

4. A square of side length 1 and a circle of radius $\frac{1}{\sqrt{3}}$ share the same center. What is the area inside the circle, but outside the square?

   (a) $\frac{2\pi}{9} - \frac{1}{\sqrt{3}}$
   (b) $\frac{\pi}{27} + \frac{1}{\sqrt{3}}$
   (c) $\frac{\pi}{3} - \frac{1}{4}$
   (d) $\frac{\pi}{3} - 1$
   (e) $1 - \frac{\pi}{9}$

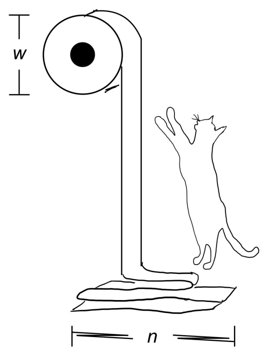
   Answer: A

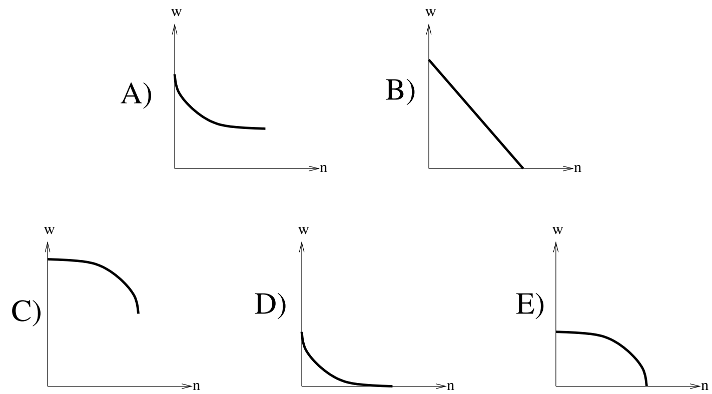
5. At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

   (a) 8
   (b) 18
   (c) 24
   (d) 28
   (e) 36

   Answer: B

6. A kitten discovers a roll of toilet paper and begins to play with it. Every time the kitten claws at the roll, she unrolls three squares. Which of the graphs below could be a graph of the width w of the roll as a function of the number of unrolled squares n?





(a) Graph A
(b) Graph B
(c) Graph C
(d) Graph D
(e) Graph E

Answer: C

7. An urn contains 5 orange balls and 6 black balls. If two balls are drawn at random without replacement, what is the probability that one is orange and the other is black?

(a) $\frac{3}{11}$
(b) $\frac{9}{22}$
(c) $\frac{6}{11}$
(d) $\frac{15}{22}$
(e) $\frac{9}{11}$

Answer: C

8. Let a_n be a sequence of positive integers with the property that, for all n, the arithmetic mean of the terms $a_1, a_2, \ldots, a_n$ is equal to n. Find a_{2024}.

   (a) 2024
   (b) 2025
   (c) 2712
   (d) 3036
   (e) 4047

   Answer: E

9. Which of these numbers is largest?

   (a) 2023×2025
   (b) 2024×2024
   (c) 2022×2026
   (d) 2020×2028
   (e) 2021×2027

   Answer: B

10. Compute the infinite sum $2024 + 552 + \dfrac{1656}{11} + \cdots + \dfrac{552 * 3^n}{11^n} + \ldots.$

    (a) 2782
    (b) 2783
    (c) 2784
    (d) 2785
    (e) 2786

    Answer: B

11. Find all numbers k so that the equation

$$\frac{x-1}{x-2} = \frac{x-k}{x-6}$$

has no solution for x.

(a) 2, 5, and 6.
(b) 6 only.
(c) 5 only.
(d) 2 and 5.
(e) 5 and 6.

Answer: E

12. An arithmetic sequence $\{a_n\}$ has $a_1 + a_2 + \cdots + a_{22} = 10$ and $a_1 + a_2 + \cdots + a_{44} = 30$. Find $a_1 + a_2 + \cdots + a_{2024}$.

(a) 42780
(b) 84640
(c) 253980
(d) 1015990
(e) 40945530

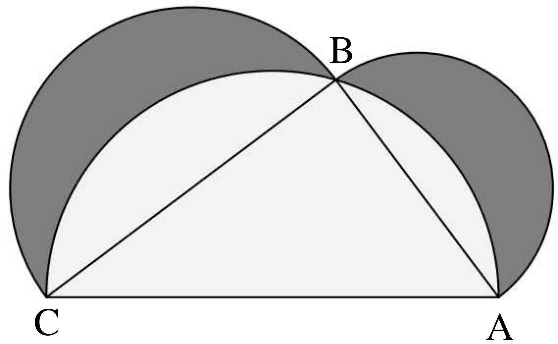
Answer:A

13. Simplify: $\dfrac{\log_4(253)}{\log_8(253)}$.

(a) 4
(b) $\dfrac{2}{3}$
(c) $\dfrac{4}{3}$
(d) $\dfrac{1}{2}$
(e) $\dfrac{3}{2}$

Answer: E

14. In triangle ABC, $\angle B = 90°$. Semicircles are constructed on sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, as shown below. If the area of triangle ABC is 2024, find the area of the shaded region.



(a) $2024\pi - 1012\sqrt{2}$

(b) 1012π

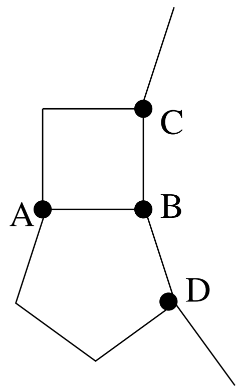
(c) $253(\pi + \sqrt{3})$

(d) $1012\pi - 506\sqrt{2}$

(e) 2024

Answer: E

15. In the picture, points A, B, and C are consecutive vertices of a square, points A, B, and D are consecutive vertices of a regular pentagon, and points C, B, and D are consecutive vertices of a regular n-gon. Find n.



(a) 12
(b) 15
(c) 18
(d) 20
(e) 24

Answer: D

16. What is the largest integer that is a divisor of the product $(n+1)(n+3)(n+5)(n+7)(n+9)$ for all positive integers n?

(a) 5
(b) 6
(c) 10
(d) 15
(e) 20

Answer: D

17. Let ABC be an isosceles triangle, with $AC = BC = 1012$. Equilateral triangles ABX, BCY and CAZ are constructed outside of $\triangle ABC$. If $CX = 2024$, find AY.

(a) $1012 + 506\sqrt{3}$

(b) $1012\sqrt{3}$

(c) $506\sqrt{3} + \dfrac{1012\sqrt{3}}{3}$

(d) 2024

(e) $1012\sqrt{6} - 506$

Answer: D

18. Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

(a) $\dfrac{3}{8}$

(b) $\dfrac{5}{12}$

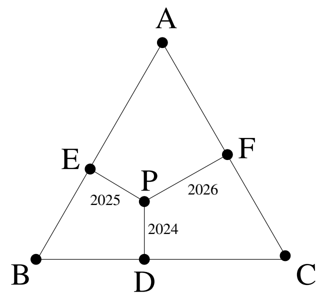
(c) $\dfrac{11}{24}$

(d) $\dfrac{1}{2}$

(e) $\dfrac{13}{24}$

Answer: A

19. ABC is an equilateral triangle, and P is a point on its interior such that the perpendicular distances from the sides BC, AB and AC are $PD = 2024$, $PE = 2025$, and $PF = 2026$ respectively. Find the length of a side of ABC.



(a) $2025\sqrt{3}$

(b) $2025\sqrt{3} - 1$

(c) 6072

(d) $\sqrt{3(2025)^2 - 1}$

(e) $4050\sqrt{3}$

Answer: E

20. Let

$$f(x) = \frac{x - \sqrt{3}}{x\sqrt{3} + 1}.$$

What is $f^{2024}(x)$?

[Here $f^{2024}(x)$ means $f(f(f(f(\ldots f(x)))))$, where the function is applied to its own output 2024 times. For example, $f^2(x) = f(f(x)) = \dfrac{\frac{x-\sqrt{3}}{x\sqrt{3}+1} - \sqrt{3}}{\frac{x-\sqrt{3}}{x\sqrt{3}+1}\sqrt{3} + 1}$.]

(a) $\dfrac{x - \sqrt{3}}{x\sqrt{3} + 1}$

(b) $2^{2024}x$

(c) $\dfrac{x + \sqrt{3}}{x\sqrt{3} + 1}$

(d) $\dfrac{-x - \sqrt{3}}{x\sqrt{3} - 1}$

(e) x

Answer: D

21. 2024 has an unusually large number of factors for an integer its size. Find the next year with exactly as many factors as 2024.

(a) 2025

(b) 2028

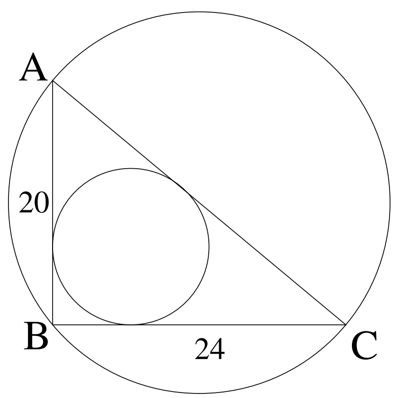
(c) 2030

(d) 2032

(e) 2034

Answer: C

22. In the picture, ABC is a right triangle with legs $AB = 20$ and $BC = 24$. The smaller circle, with diameter d, is inscribed in ABC. The larger circle, with diameter D, is circumscribed about ABC. Find $d + D$.



(a) $4\sqrt{11} + 4\sqrt{61}$

(b) $4\sqrt{61} + \dfrac{60}{11}$

(c) 44

(d) $\dfrac{960}{44 + 4\sqrt{61}}$

(e) $4\sqrt{61}$

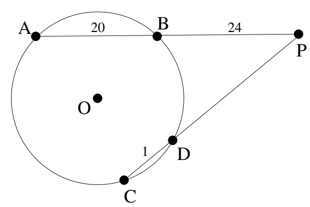
Answer: C

23. What are the last three digits of 2024^{2024}?

(a) 024
(b) 576
(c) 776
(d) 824
(e) 976

Answer: C

24. In the picture, chords AB and CD of circle O meet outside the circle at P. If $AB = 20$, $BP = 24$, and $CD = 1$, find DP.



(a) 32
(b) 33
(c) 34
(d) 35
(e) 36

Answer: A

25. Let $S = 1!2! \cdots 100!$. How many positive integers k are there such that $\dfrac{S}{k!}$ is a perfect square?

(a) none
(b) 1
(c) 2
(d) 4
(e) 8

Answer: B