- 1. 2023 is to be factored as a product of three positive integers, 2023 = abc with a < b < c. How many possible values could b take?
 - (a) 0
 - (b) 6
 - (c) 4
 - (d) 7
 - (e) 2

Answer: E.

- 2. Find the difference between O, the sum of the positive odd numbers less than or equal to 2023, and E, the sum of the positive even numbers less than 2023.
 - (a) 0
 - (b) 1
 - (c) 2
 - (d) 1011
 - (e) 1012

Answer: E.

- 3. A ball falls into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area, A, of the circle in terms of the time, t (in seconds), that has passed since the ball hit the lake.
 - (a) $A(t) = 25\pi t \text{ cm}^2$
 - (b) $A(t) = \pi t^2 \text{ cm}^2$
 - (c) $A(t) = 25\pi t^2 \text{ cm}^2$
 - (d) $A(t) = 5\pi t^2 \text{ cm}^2$
 - (e) $A(t) = 10\pi t \text{ cm}^2$

Answer: C

- 4. A **Pythagorean triple** is a list of three positive integers 0 < a < b < c with the property that $a^2 + b^2 = c^2$. (Some examples of Pythagorean triples are (3, 4, 5), (6, 8, 10), and (8, 15, 17).) Which of the following statements must be true for a Pythagorean triple (a, b, c)?
 - I: a+b > c.
 - II: a + b < c.
 - III: $a^3 + b^3 > c^3$.
 - IV: $a^3 + b^3 < c^3$.
 - (a) I only.
 - (b) II and III.
 - (c) II and IV.
 - (d) None of these.
 - (e) I and IV.

Answer: E

- 5. If five cowboys can mow six lawns in seven hours and seven sooners can mow five lawns in six hours, how many lawns can a team of two cowboys and three sooners mow in forty hours?
 - (a) 5
 - (b) 17
 - (c) 25
 - (d) 28
 - (e) 30

Answer: D.

- 6. What is the last digit of 2023^{2023} ?
 - (a) 1
 - (b) 3
 - (c) 5
 - (d) 7
 - (e) 9

Answer: D.

7. A fountain fills a spherical bottle with water at a constant rate of 1 gallon per minute. Which of the following graphs could represent the height of the water in the bottle as a function of the volume of water in the bottle?



- (c) Graph III.
- (d) Graph IV.
- (e) Graph V.

Answer: E

8. Which of the numbers below is the largest?

(a)
$$\frac{10^{2020} - 1}{10^{2021} - 1}$$

(b)
$$\frac{10^{2021} - 1}{10^{2022} + 1}$$

(c)
$$\frac{10^{2022} + 1}{10^{2023} + 1}$$

(d)
$$\frac{10^{2023} + 1}{10^{2024} - 1}$$

(e)
$$\frac{10^{2024} - 1}{10^{2025} - 1}$$

Answer: C.

9. Find the center of the circle $x^2 + y^2 + 7x - 17y = 2023$.

(a)
$$\left(-\frac{7}{2}, -\frac{17}{2}\right)$$

(b) $\left(-\frac{17}{2}, \frac{7}{2}\right)$
(c) $\left(-\frac{17}{2}, -\frac{7}{2}\right)$
(d) $\left(\frac{17}{2}, \frac{7}{2}\right)$
(e) $\left(-\frac{7}{2}, \frac{17}{2}\right)$



10. Find the remainder when $x^{10} - 10$ is divided by $x^2 - 2$.

- (a) 0
- (b) 5
- (c) 22
- (d) 50
- (e) 96

Answer:C.

11. In square ABCD, AE = 9 and EB = 3. If DF = FA and FG is perpendicular to DE, find CG.



- (a) 4
- (b) 5
- (c) 6
- (d) 7
- (e) 8

Answer: A

12. A student shows up to school and claims that the dog ate her math homework. As it turns out, this is true, and the page is half chewed up. The problem seems to say

Find the roots of the polynomial

 $x^4 - 5x^3 - 8x^2 +$ (remainder is illegible)

The student's work is also torn up, but it looks as if her first three solutions are x = -2, x = 0, and x = 1. Assuming that her work was correct, what is the missing solution?

- (a) -5
- (b) -3
- (c) 3
- (d) 6
- (e) 9

Answer: D

13. If
$$y = \frac{2x}{2x+3}$$
, what is x as a function of y?
(a) $\frac{3y}{2y+2}$
(b) $-\frac{2y-2}{3y}$
(c) $-\frac{3y}{2y-2}$
(d) $-\frac{3y-2}{2y}$
(e) $\frac{3y}{2y-2}$



- 14. Charlotte's piggy bank contains 100 coins, all of which are nickels, dimes, and quarters. If Charlotte's quarters were nickels and her nickels were quarters, the total value of her piggy bank would increase by \$2.40. If Charlotte's quarters were dimes and her dimes were quarters, the total value would increase by \$1.95. How much money does Charlotte have?
 - (a) \$11.85
 - (b) \$11.90
 - (c) \$11.95
 - (d) \$12.00
 - (e) \$12.05

Answer:B

15. In the figure, AB is a diameter of semicircle O, which has radius 1. If BC = 1 and CD is parallel to AB, find the area of the shaded region.

B





16. Recall that, for a real number t, the expression $\lfloor t \rfloor$ represents the "floor" of t, that is, the greatest integer less than or equal to t. For example, $\lfloor 2.9 \rfloor = 2$ and $\lfloor 3 \rfloor = 3$.

For how many positive integers $x \le 2023$ is $\lfloor \log x \rfloor$ even? (Recall that "log" stands for the logarithm base ten.)

- (a) 909
- (b) 1010
- (c) 1111
- (d) 1212
- (e) 1313

Answer:A.

- 17. An integer N is chosen at random between 1 and 20 (inclusive). What is the probability that N + 1 is a divisor of $N^2 + 4N + 23$?
 - (a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) $\frac{7}{20}$ (d) $\frac{2}{5}$ (e) $\frac{9}{20}$

Answer:A.

18. Define a sequence by the rule $x_1 = \frac{5}{4}$ and $x_{n+1} = \frac{16x_n^2 - 21}{16}$ for all *n*. Compute x_{2023} .

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $-\frac{5}{4}$ (d) $-\frac{21}{16}$ (e) $\frac{5}{4}$

Answer: C

19. A spider is at point A of its web (shown below) when it captures a fly at point B. It then travels to point B along the strands of its web. Assuming the spider only moves from left to right along the horizontal strands, and only upwards along the diagonal strands (with either slope), how many distinct paths could it take?



- 20. A False Fibonacci sequence is a sequence of positive integers $\{f_1, f_2, f_3, ...\}$ in which $f_{n+2} = f_{n+1} + f_n$ for all n (but f_1 and f_2 are not necessarily equal to 1). If $f_1, f_2, ...$ is a false Fibonacci sequence and $f_6 = 20$, find f_7 .
 - (a) 31
 - (b) 32
 - (c) 33
 - (d) 34
 - (e) 35

Answer:C

21. In the figure, triangles ABD, BCE, and CAF are equilateral, and points D, A, and F are collinear. If triangle ABD has area 36, triangle BCE has area 43, and triangle CAF has area 49, find the area of triangle ABC.



- (a) 42
- (b) 43
- (c) 44
- (d) 45
- (e) 46

Answer: A.

- 22. A desk contains pages which are labeled with positive integers. A robot tidies the desk according to the following instructions:
 - If there are papers on the desk, choose one at random and read its label. Then:
 - If the chosen paper was labeled with the number N, replace it on the desk with N 1 new pages, labeled with the numbers from 1 to N 1.
 - Recycle the chosen paper.
 - If there are no papers on the desk, move on to the next office.

When the robot arrives, the desk contains a single page, labeled "2023". How many pages does the robot recycle before moving on?

- (a) 2022!
- (b) 2023
- (c) 2,047,276
- (d) 4,092,529
- (e) 2^{2022}

Answer: E.

23. Find the sum of the squares of the roots of the polynomial $x^3 + 6x^2 + 8x + 2$.

- (a) 12
- (b) 14
- (c) 16
- (d) 18
- (e) 20

Answer: E

24. In the figure, AB, BC, and CD are all tangent to circle O, and AB is parallel to CD. If AB = 20 and CD = 23, find the area of circle O.



- (a) 400π
- (b) 415π
- (c) 430π
- (d) 445π
- (e) 460π

Answer: E

25. Simplify $\frac{\cos 23^{\circ} + \sin 23^{\circ}}{\cos 23^{\circ} - \sin 23^{\circ}}$. (a) $\cot 22^{\circ}$ (b) $\csc 23^{\circ}$ (c) $\sec 24^{\circ}$ (d) -1(e) $\tan 46^{\circ}$ Answer: A