1. Express the area of a square in terms of its perimeter.

   A. $4\sqrt{p}$

   B. $16p^2$

   C. $\dfrac{p^2}{16}$

   D. $4p^2$

   E. $\dfrac{p^2}{4}$

   Answer: C

2. Alice flips a coin ten times and records the results as an ordered list. What is the probability that, when Bob repeats this experiment, exactly one of the listed results matches up? For example,

   TTHTHHTH**T**H and
   HHTHTTHT**T**T

   would fit this criterion, but neither

   TT**H**THHTH**T**H and
   HH**H**HTTHT**T**T

   nor

   TTHTHHTHHH and
   HHTHTTHTTT

   would.

   A. $\dfrac{1}{1024}$

   B. $\dfrac{5}{512}$

   C. $\dfrac{1}{10}$

   D. $\dfrac{25}{128}$

   E. $\dfrac{2835}{8192}$

   Answer: B.

3. What is the sum of the prime factors of 2018?

   A. 75

   B. 1011

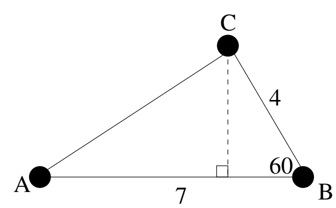
   C. 1012

   D. 2019

   E. 3030

   Answer: B.

4. In triangle ABC, side AB has length 7, side BC has length 4, and angle B has measure $60°$. Find the length of side AC.



   A. 6

   B. $\sqrt{37}$

   C. $4\sqrt{3}$

   D. $\sqrt{61}$

   E. 8

   Answer: B.

5. For how many natural numbers n is the inequality

$$2^n > n!$$

   true? [0 is not a natural number.]

   A. None.

   B. One.

   C. Two.

   D. Three.

   E. Infinitely many.

   Answer: D

6. In trapezoid $ABCD$, sides AB and CD are parallel. The diagonals are perpendicular with $AC = 9$ and $BD = 12$. Find the area of $ABCD$.

   A. 27

   B. 36

   C. $27\sqrt{3}$

   D. 54

   E. 108

   Answer: D.

7. How many integers n make the inequality

$$\frac{(4^2-2^2)(5^2-2^2)}{n^2-2^2} < \frac{(2)(3)(4^2-3^2)(5^2-3^2)}{n^2-3^2}$$

   true?

   A. It is true for all n.

   B. It is true for infinitely many n, and false for finitely many n.

   C. It is true for infinitely many n, and false for infinitely many n.

   D. It is false for infinitely many n, and true for finitely many n.

   E. It is false for all n.

   Answer: B

8. A real number x is equal to the number obtained from x by taking its reciprocal, then shifting the decimal point three places to the right. How many possible values could x have?

   A. None.

   B. One.

   C. Two.

   D. More than two, but finitely many.

   E. Infinitely many.

   Answer: C

9. The area of a circle (measured in square feet) is equal to its circumference (measured in inches). Find its radius. [Circles have positive area.]

   A. $12\sqrt{\pi}$ feet.

   B. 24 feet.

   C. 12π feet.

   D. $\dfrac{144}{\pi}$ feet.

   E. 144 feet.

   Answer: B

10. How many ordered pairs of positive integers (n, k) make the number $p = 7^n + 3^k$ prime?

    A. None.

    B. One.

    C. Two.

    D. More than two, but finitely many.

    E. Infinitely many.

    Answer: A.

11. Say that a nine-digit positive integer n is "fun" if it satisfies the two properties:

    - Each of the digits from 1 to 9 appears exactly once in the decimal representation of n.
    - n is divisible by 15.

    How many integers are fun?

    A. 12096

    B. 24192

    C. 40320

    D. 72576

    E. 362880

    Answer: C

12. A triangle is contained in a square of side length 2018, and its area is as large as possible. What is its area?

    A. $\dfrac{(2018)(2017)}{2}$

    B. $\dfrac{2018^2}{2}$

    C. 2018^2

    D. $\dfrac{2018^2\sqrt{6}}{4}$

    E. $\dfrac{2018^2\sqrt{3}}{4}$

    Answer: B

13. For how many integers n does the polynomial $f(x) = x^2 - nx - 2018$ have at least one rational root?

    A. None.

    B. One.

    C. Two.

    D. More than two, but finitely many.

    E. Infinitely many.

    Answer: D.

14. Suppose $\{a_n\}$ is a geometric sequence with the property that every term is an integer power of 2018. If $\log_{2018}(a_2) = 5$ and $\log_{2018}(a_3a_4a_5) = 27$, find $\log_{2018}(a_7)$.

    A. 10

    B. 11

    C. 12

    D. 15

    E. 20

    Answer: D.

15. Suppose a, b, c, and d are distinct integers with the property that $abcd = 2018$. How many different possible values could the sum $a + b + c + d$ take?

    A. None. (No such a, b, c, and d exist).

    B. One.

    C. Two.

    D. More than two, but finitely many.

    E. Infinitely many.

    Answer: C

16. A right circular cone with height 20 and base radius 10 stands on its vertex and is filled with water to a height of 18. Then the base is sealed and the cone is turned over, so that it stands on its base. How high does the water go?

    A. Between 4 and 6.

    B. Between 6 and 8.

    C. Between 8 and 10.

    D. Between 10 and 12.

    E. Between 12 and 14.

    Answer: B

17. What are the last two digits of 2018^{2018}?

    A. 76

    B. 18

    C. 68

    D. 24

    E. 32

    Answer: D.

18. Compute $\sum_{n=1}^{n=2018} \lfloor \sqrt{n} \rfloor$. (Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

A. 59462

B. 59463

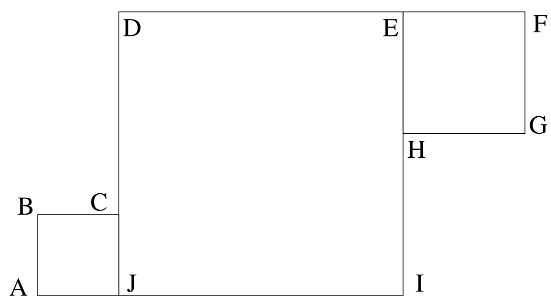
C. 59464

D. 59465

E. 59466

Answer: E

19. In the figure below, $ABCJ$, $DEIJ$, and $EFGH$ are all squares, and C lies on DJ and H lies on EI. If $DE = 18$, find the area of quadrilateral $BDGI$.



A. 324

B. $324 + 8\sqrt{2}$

C. 339

D. $324 + 12\sqrt{6}$

E. $324 + 8\sqrt{2} + 12\sqrt{6}$

Answer: A

20. Two parents and five children sit in a row of seven seats at Boone Pickens Stadium, and each parent must sit between two children. In how many ways can the group arrange itself to satisfy this condition? (The children are all different from one another.) (The parents are also different from each other.)

  A. 120

  B. 720

  C. 1440

  D. 2400

  E. 2520

  Answer: C

21. Three circles of radius one are arranged so that each is externally tangent to the other two. A fourth, larger, circle is internally tangent to all three of the smaller circles. Find its radius.

  A. $\dfrac{3+2\sqrt{3}}{3}$

  B. $\dfrac{4\sqrt{3}-2}{3}$

  C. $\dfrac{4\sqrt{3}}{3}$

  D. $\dfrac{3+4\sqrt{3}}{3}$

  E. 2

  Answer: A

22. Consider the set of all numbers R $(0 \leq R < 2018^2)$ that occur as the remainder when $2019^n - 2017^n$ is divided by 2018^2 (for nonnegative integers n). How many elements are in this set?

  A. 2

  B. 4

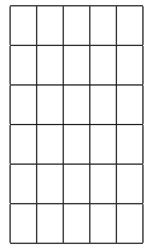
  C. 1010

  D. 2017

  E. 2018^2

  Answer: C

23. How many rectangles are in the 6×5 grid below? [Rectangles have positive area.]



A. 315
B. 320
C. 325
D. 330
E. 335

Answer: A

24. At how many points in the interior of the first quadrant do two or more of the graphs of the equations

$$y = e^x, \qquad y = x^e, \qquad x = e^y, \qquad x = y^e, \qquad e = x^y, \qquad e = y^x$$

intersect? [The origin is not in the interior of the first quadrant.]

A. 9
B. 10
C. 12
D. 14
E. 15

Answer: C

25. Trapezoid $ABCD$ has right angles at B and C. If $AB = 1$, $CD = 4$, and the circle with diameter AD is tangent to BC, find the area of $ABCD$.

A. 7
B. 8
C. 9
D. 10
E. 11

Answer: D