Part I. Individual Round

1. What is the largest possible number of intersections between a circle and a parabola?

   A. 2

   B. 4

   C. 6

   D. 8

   E. 10

   Answer: B.

2. Twelve cowboys sit in a circle around a bonfire. Each observes that his age (viewed as an integer) is the average of the ages of his left and right neighbors. Which of the following could be the sum of their ages?

   A. 224

   B. 225

   C. 226

   D. 227

   E. 228

   Answer: E

3. How many distinct solutions does the equation $|||x-1|-1|-1| = 2013$ have?

   A. None

   B. One

   C. Two

   D. Four

   E. Eight

   Answer: C

4. Roquentin has six books, two of which are by Sartre. How many ways can Roquentin arrange his books on a shelf, assuming the two books by Sartre must always be next to one another?

   A. 48

   B. 120

   C. 240

   D. 360

   E. 720

   Answer: C

5. What is the remainder when $x^3 - 2x^2 - 1$ is divided by $x + 2$?

   A. -15

   B. 15

   C. -1

   D. -17

   E. 17

   Answer: D

6. Pieces in the shape of an equilateral triangle of side length 10 cm are used to tile a mosaic in the shape of an equilateral triangle with side length 2 meters. How many such pieces are needed?

   A. 20

   B. 40

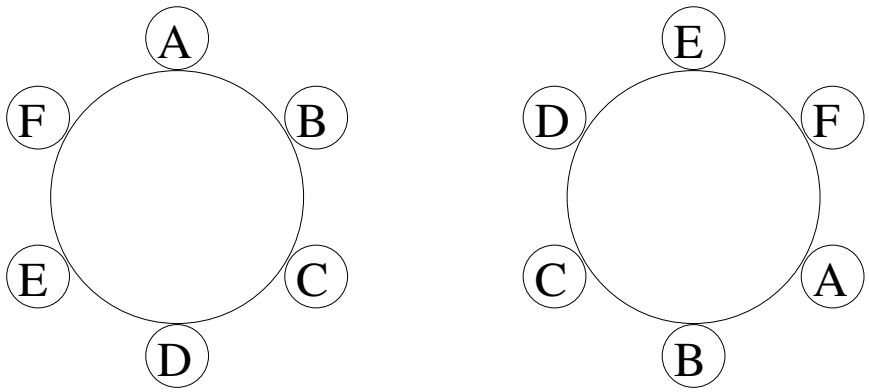
   C. 200

   D. 400

   E. 720

   Answer: D

7. Three married couples sit around a round table. In how many ways can they be seated if each person must be directly opposite his/her spouse? (We count seating arrangements *up to rotation*. That is, if one seating arrangement can be rotated into another, we consider them to be the same seating arrangement. For example, the two arrangements below are the same.)



A. 3

B. 5

C. 6

D. 8

E. 12

Answer: D

8. Let S be the set of points that are exactly twice as far from $(2, 0)$ as they are from $(1, 3)$. Find the equation of S.

A. $9x^2 + y^2 - 6xy + 11x - 33y + 18 = 0$

B. $x^2 + 9y^2 - 6xy + 39x - 107y + 318 = 0$

C. $3x - 9y + 14 = 0$

D. $3x^2 + 3y^2 - 4x - 24y + 36 = 0.$

E. $3x + y - 6 = 0$

Answer: D

9. Simplify the expression

$$\log_2 6 + \log_4 18 - \log_2 9.$$

A. $\frac{3}{2}$

B. 0

C. $\log_2 15$

D. $\log_4 12$

E. $\log_2 6$

Answer: A

10. What is the sum of all odd two-digit numbers? (One-digit numbers are *not* two-digit numbers.)

A. 2475

B. 2500

C. 2525

D. 2550

E. 2575

Answer: A

11. Find the period of the function $y = \cos(6x) + \cos(9x)$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. 2π

E. 36π

Answer: C

12. Find the distance between the circles $(x-10)^2+(y+4)^2 = 169$ and $(x+5)^2+(y-4)^2 = 1$.

    A. 3

    B. 4

    C. 5

    D. 6

    E. 7

    Answer: A

13. What is the sum of the positive divisors of 2013? (1 and 2013 are divisors of 2013.)

    A. 2014

    B. 2688

    C. 2760

    D. 2793

    E. 2976

    Answer: E

14. Find the distance from the point $(1, 2)$ to the line $3x + 4y = 5$.

    A. $\frac{\sqrt{6}}{5}$

    B. $\frac{6}{5}$

    C. $\frac{11}{5}$

    D. 6

    E. $\frac{6\sqrt{5}}{5}$

    Answer: B

15. How many pairs of positive integers (x, y) satisfy $20x + 13y = 2013$?

    A. 4

    B. 8

    C. 13

    D. 33

    E. 100

    Answer: B

16. If $\sin a + 8 \sin b + 15 \cos b = 18$, then what is the absolute value of $\cos a$?

A. 0
B. $\frac{2}{3}$
C. $\sqrt{\frac{8}{15}}$
D. $\sqrt{\frac{15}{18}}$
E. 1

Answer: A

17. Find the maximum possible value of $x + y$ on the region in the first quadrant defined by the inequalities $x + 2y \leq 6$ and $4y + 5x \leq 20$.

A. 4
B. $\frac{49}{12}$
C. $\frac{25}{6}$
D. $\frac{17}{4}$
E. $\frac{13}{3}$

Answer: E

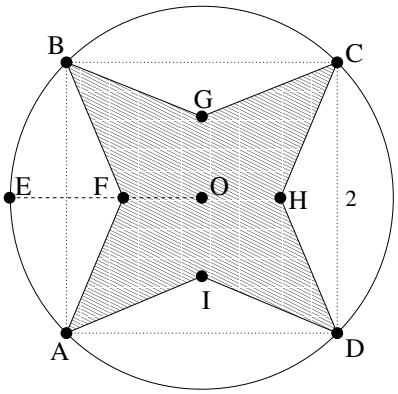
18. What is the largest root of the equation

$$(1 - x)^2 + (2 - x)^2 + \cdots + (2013 - x)^2 = 671 \cdot 1007 \cdot 4027?$$

A. $\frac{4027 + \sqrt{1007}}{671}$
B. This equation has no real roots.
C. 2014
D. 0
E. $\frac{-671 + \sqrt{1007 \cdot 4027}}{2}$

Answer: C

19. In the figure below, square $ABCD$ has sides of length 2 and is inscribed in circle O. E, F, and O are collinear, and EF and AB are perpendicular bisectors. Assuming star $AFBGCHDI$ has 90° rotational symmetry, what is its area?



A. $1 + 2\sqrt{2}$

B. $2\sqrt{2}$

C. $2 + \sqrt{2}$

D. $8 - 4\sqrt{2}$

E. $2\sqrt{2} - 2$

Answer: D

20. If $x^2 - x + 1 = 0$, then what is $x^{2013} - x^{671} + x^{11}$?

A. 0

B. $2013 - 1353\sqrt{3}$

C. $\dfrac{1353\sqrt{3}}{2}$

D. 3

E. -1

Answer: E

21. A regular dodecagon is inscribed in a circle of radius 1. What is the area outside the dodecagon but inside the circle? (A dodecagon has twelve sides.)

A. $\frac{\pi}{12}$

B. $\pi - \frac{5\sqrt{6}}{4}$

C. $\pi - \frac{22}{7}$

D. $\pi - \frac{3\sqrt{3}}{2}$

E. $\pi - 3$

Answer: E

22. What is the largest integer n such that 2^n divides the product

$$P = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot 2013?$$

A. 2000

B. 2001

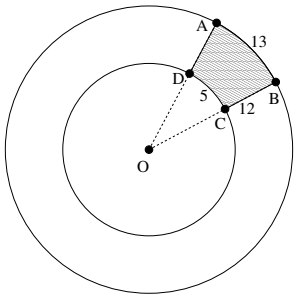
C. 2002

D. 2003

E. 2004

Answer: E

23. In the figure below, the two circles are concentric with shared center O. A, D, and O are collinear, as are B, C, and O. Given that $\overset{\frown}{\mathrm{AB}} = 13$, $\overline{BC} = 12$, and $\overset{\frown}{\mathrm{CD}} = 5$, find the area of the shaded region.



A. 30

B. 30π

C. 78

D. $\dfrac{390}{\pi}$

E. 108

Answer: E

24. Bob, Mack, and Mike share ownership of a ping-pong table. They draw straws to decide who plays the first game, then after each game the loser sits out while the winner plays against the previous sit-out. When they are done, Mike has played 15 games and Bob has played 7. How many games were played in total?

A. 8

B. 15

C. 17

D. 19

E. 22

Answer: B

25. Let $ABCD$ be a quadrilateral inscribed in a circle of radius 25, such that $AB = 14$, $BC = 48$, and $AC \perp BD$. Find BD.

A. $\frac{1415}{48}$

B. $\frac{977}{35}$

C. $\frac{672}{25}$

D. $\frac{387}{14}$

E. $\frac{121}{9}$

Answer: C