Math 5613

Assignment 14

Due Friday, December 5

Part one: Reading. Read through Chapter 14.8 in the textbook (more or less: see the "homework" page on the course web page for precise daily goals).

Part two: Problems to solve and write up.

I want your effort on both the problem-solving and the writeup to be collaborative. This week, you'll again be responsible for turning in writeups of three problems, but you'll also have to arrange to meet with me on Friday or early in the next week to present another orally.

Throughout, assume unless otherwise indicated that rings are commutative with 1, and that notation remains intuitive: $F \subset K$ are fields, $n \in \mathbb{Z}$, etc.

1. Let $f = x^n - a \in \mathbb{Q}[x]$ be irreducible, and suppose $a > 0$. If K is the splitting field of f, determine $[K : \mathbb{Q}]$.

   [The answer depends on n, of course, but also on the parity of n. To see why the parity matters, consider that positive real numbers have real square roots.]

2. Let $f = x^6 - 2x^3 - 2 \in \mathbb{Q}[x]$, and set K equal to the splitting field of f and $F = \mathbb{Q}(i, \sqrt{3}, \sqrt[3]{2})$. Let $\sigma \in \mathrm{Gal}(K/\mathbb{Q})$ have order 3. What is the fixed field of $\sigma\,|_F$? Under the (false) assumption that $F = K$, what are the possible cycle decompositions of σ, when it acts on the roots of f?

3. Let $f = x^6 - 2x^3 - 2 \in \mathbb{Q}[x]$, and set K equal to the splitting field of f. Given that $[K : \mathbb{Q}] = 36$, describe all elements of $G = \mathrm{Gal}(K/\mathbb{Q})$, and find the isomorphism type of G.

4. Find the Galois group of $x^{15} + 15x + 12$ over $\mathbb{Q}$.

Part three: Estimate the time you spent on this assignment. I will pay attention to this in writing future assignments. Meanwhile, if it's taking you longer than you think is reasonable, please talk to me so we can come up with a strategy.