Some worked out problems for exam 2 review

Beware of typos :)

Q: Compute $\int 2x^3 e^{x^2}\,dx$

A: First do substitution $x^2 = w$ so $dw = 2x\,dx$ so
$\int 2x^3 e^{x^2}\,dx = \int 2x(x^2)e^{x^2}\,dx = \int we^w\,dw$

Now integration by parts
$\int we^w\,dw = we^w - \int e^w\,dw = we^w - e^w + C$

Now put $w = x^2$ back
$we^w - e^w + C = x^2e^{x^2} - e^{x^2} + C$

Q: Does $\sum_{n=1}^{\infty} n^{-20}$ converge?

A: Yes, it is a p-series with $p = 20 > 1$ that is
$\sum_{n=1}^{\infty} n^{-20} = \sum_{n=1}^{\infty} \frac{1}{n^{20}}$

Q: Compute $\int_2^{\infty} \frac{2}{x^3}\,dx$

A: $\int_2^{\infty} \frac{2}{x^3}\,dx = \lim_{R\to\infty} \int_2^{R} \frac{2}{x^3}dx = \lim_{R\to\infty} \left[\frac{-1}{x^2}\right]_{x=2}^{R} = \lim_{R\to\infty} \left(\frac{-1}{R^2} - \frac{-1}{2^2}\right) = \frac{1}{4}$

Q: Compute $\int_1^{2} \frac{1}{x^2(x-3)}\,dx$

A: First do partial fractions:

$$\frac{1}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$= \frac{Ax(x-3) + B(x-3) + Cx^2}{x^2(x-3)} = \frac{(A+C)x^2 + (B-3A)x - 3B}{x^2(x-3)}$$

So $A + C = 0$, $B - 3A = 0$, and $-3B = 1$. Therefore $C = -A$, $A = B/3$ and $B = -1/3$. Thus $A = -1/9$ and $C = 1/9$ and so

$$\frac{1}{x^2(x-3)} = \frac{-1/9}{x} + \frac{-1/3}{x^2} + \frac{1/9}{x-3}$$

$$\int_1^{2} \frac{1}{x^2(x-3)}\,dx = \int_1^{2} \frac{-1/9}{x} + \frac{-1/3}{x^2} + \frac{1/9}{x-3}\,dx$$

$$= \left[-(1/9)\ln|x| + \frac{1}{3x} + (1/9)\ln|x-3|\right]_{x=1}^{2}$$
$$= -(1/9)\ln 2 + \frac{1}{6} + (1/9)\ln|2-3| + (1/9)\ln 1 - \frac{1}{3} - (1/9)\ln|1-3|$$
$$= -(2/9)\ln 2 - \frac{1}{6}$$

Q: What are the poles of $\dfrac{x^7}{x(x-1)}$

A: not a proper fraction so ∞ is a pole. So is $x = 1$ and $x = 0$ (denominator vanishes).

Q: Compute T_2 for $\int_0^1 x^2\,dx$

A:

$x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1$ $y_0 = x_0^2 = 0$ $y_1 = x_1^2 = \frac{1}{4}$ $y_2 = x_2^2 = 1$

$\Delta x = \frac{1}{2}$

$T_2 = \frac{1}{2}\Delta x\,(y_0 + 2y_1 + y_2) = \frac{1}{2}\frac{1}{2}\left(0 + 2\frac{1}{4} + 1\right) = \frac{3}{8}$

Q: Compute $\lim\limits_{n\to\infty} \dfrac{2n^2}{3n^2 + \sqrt{n}}$

A: Change to a continuous variable and apply l'Hospital's rule

$$\lim_{n\to\infty} \frac{2n}{3n + \sqrt{n}} = \lim_{x\to\infty} \frac{2x}{3x + \sqrt{x}} = \lim_{x\to\infty} \frac{2}{3 + \frac{1}{2\sqrt{x}}} = \frac{2}{3}$$

Q: Compute $\displaystyle\int \cos^2 2x\,dx$ using the complex exponential.

A:

$$\int (\cos 2x)^2\ dx = \int \left(\frac{e^{i2x} + e^{-i2x}}{2}\right)^2\ dx = \int \frac{1}{4}\left(e^{i4x} + 2e^0 + e^{-i4x}\right)\ dx$$
$$= \frac{1}{4}\frac{1}{4i}e^{i4x} + \frac{2x}{4} + \frac{1}{4}\frac{1}{-4i}e^{-i4x} + C$$
$$= \frac{x}{2} + \frac{1}{8}\frac{e^{i4x} - e^{-i4x}}{2i} + C$$
$$= \frac{x}{2} + \frac{\sin 4x}{8} + C$$