EXERCISE SET # 4

Exercise 1: For $\mathcal{L}$ a framed link in $S^3$, prove that the linking matrix of $\mathcal{L}$ is a presentation matrix for $H^2(S^3(\mathcal{L}); \mathbb{Z}) \cong H_1(S^3(\mathcal{L}); \mathbb{Z})$.

Exercise 2: For $R$ a commutative ring with identity, call a 3-manifold $Y$ an $R$–homology $S^3$ ($RHS^3$) if $H_*(Y; R) \cong H_*(S^3; R)$.

- Prove that any $(\mathbb{Z}/2\mathbb{Z})HS^3$ is orientable.
- Let $Y$ be a $\mathbb{Q}HS^3$, and let $A$ and $B$ be simply-connected 4-manifolds with $\partial A = -\partial B = Y$, and form the closed 4-manifold $X = A \cup_Y B$. Show that $Q_A \oplus Q_B \hookrightarrow Q_X$, and that if $A$ and $B$ are both negative definite then so is $X$.

Exercise 3: Prove that neither of the following lens spaces bound a $\mathbb{Q}HB^4$.

- $L(25,18)$
- $L(2,1)$

Exercise 4: Certify that the trace of surgery on the following framed link yields a negative definite 4-manifold with boundary the Poincaré homology sphere $\mathcal{P}$. Prove that $\mathcal{P}$, with this orientation, does not bound a positive-definite 4-manifold.

Exercise 5: Construct a cobordism $X: \mathcal{P} \to L(5,4)#L(3,2)#L(2,1)$.

Exercise 6: Prove that the total space of the disc bundle associated to the tangent bundle of $S^2$ is diffeomorphic to the trace of $+2$-surgery on the unknot in $S^3$ (up to orientation).