Exercise Set #1

**Exercise 1:** Let $D$ denote a diagram for a knot $K$ in $S^3$. Show that the framing coefficient of the blackboard framing in $D$ equals the writhe $w(D)$, the signed number of self-crossings of $D$.

**Exercise 2:** Let $T_{p,q}$ denote the $(p,q)$-torus knot. Recall that the exterior $T_{p,q}$ in $S^3$ is a Seifert fibered space of type $\mathbb{D}(|p|,|q|)$. Let $F$ be the isotopy class of a Seifert fiber in $\partial N(T_{p,q})$. Show that the framing coefficients of $F$ is $pq$.

**Exercise 3:** Let $(M,\xi)$ be a contact 3-manifold. A **Legendrian link** $L$ in $(M,\xi)$ is a link in $M$ whose tangent vectors all lie in $\xi$. The **canonical framing** on $L$ is the framing induced by any vector field on $L$ transverse to $\xi$.

i Discuss why any link $(S^3,\xi_c)$, where $\xi_c$ is the standard contact structure $\ker(dz + xdy)$, is isotopic to a Legendrian link. (*Hint: Introduce cusps.*)

ii Let $D$ be a Legendrian link diagram of $K$ in the standard structure on $(S^3,\xi_c)$. The Thurston-Bennequin invariant $tb(K)$ of a Legendrian knot $K$ is defined to be the canonical framing of $K$ in the standard structure $(S^3,\xi_c)$. Show that

$$tb(K) := w(D) - \lambda(D)$$

where $\lambda(D)$ is the number of left cusps.

**Exercise 4:** Let $Y$ be +1 surgery on the right-handed trefoil in $S^3$. Compute $\pi_1(Y)$.

![Diagram of Legendrian link](image)

**Exercise 5:** Prove that surgery on each the framed links in Figure 3.27 yields $\Sigma(2,3,5)$.

![Figure 3.27](image)

**Exercise 6:** Which lens space is +5 surgery on RHT?