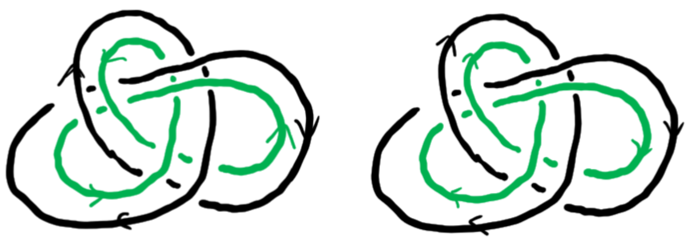
Exercise Set #5

Exercise 1: Suppose F is a surface of genus g obtained by Seifert's algorithm on a regular projection of a link of n components, c crossings and s the number of Seifert circles. Show that

$$g = 1 - \frac{s+n-c}{2}.$$

Exercise 2: Find Seifert surfaces for the following links.



(a) (b)

Exercise 3: Let K_1 and K_2 be knots in S^3 with Seifert surfaces F_1 and F_2. Denote the algebraic intersection of A and B by $A \cdot B$.

(a) Show that $F_1 \cdot K_2 = F_2 \cdot K_1$.

(b) Show that K_1 has a Seifert surface F_1' which is disjoint from K_2 if and only if $F_1 \cdot K_2 = 0$.

Exercise 4: Complete the following boundary link exercises.

a Let U be an unknot in S^3, K a knot in M_U. and $\pi : \tilde{M_U} \to M_U$ an n-fold covering map. Show that if link $L = U \cup K$ is a boundary link, then K bounds a surface F in M_U such that $\pi^{-1}(F)$ is n disjoint copies of F.

b Determine which of the following are boundary links.

(i) (ii) (iii) (iv)

Exercise 5: Let K be a knot in a $\mathbb{Z}HS$, M, and let $i : \partial M_K \to M_K$ be the inclusion map. Given a class $[p_0] \in H^1(\partial M_K)$, use exact sequences on homology and cohomology and Poincaré-Lefschetz duality to show that $[p_0] \in \mathsf{im}\, i^*$.