Exercise Set #3

Exercise 1: Two knots $K$ and $K'$ are \emph{ambient isotopic} if there is a smooth isotopy $F_t : M \to M$ such that $F_0 = id_M$ and $F_1(K) = K'$.

(a) Prove that if $M = S^3$ then $K$ is ambient isotopic to $K'$ if and only if there is an orientation preserving automorphism $f : M \to M$ such that $f(K) = K'$.

(b) Give an example of two knots in a closed orientable manifold $M$ that are equivalent but not ambient isotopic.

Exercise 2: Draw 2 different integral surgery descriptions of $L(7, 3)$.

Exercise 3: Let $K$ be a knot in $S^3$. Compute $H_1(S^3_{\mu/\nu}(K))$.

Exercise 4: Regard $D^2$ as $\{(x, y) | x^2 + y^2 \leq 1\}$. Let $\varphi : D^2 \to D^2$ be rotation about the origin by $2\pi/n$, where $n$ is a positive integer. Let $E$ be a small disk centered at $(1/2, 0)$, small enough so that $E, \varphi(E), \ldots, \varphi^{n-1}(E)$ are disjoint. Define

$$D^n := D^2 \setminus \bigcup_{i=0}^{n-1} \varphi^i(\text{int } E),$$

and define

$$X_n := \frac{D_n \times I}{(x, 0) \sim (\varphi(1), 1)}.$$

Describe $X_n$ as a link exterior, and compute $\pi_1(X_n)$.

Exercise 5: Let $H$ and $H'$ be genus 2 handlebodies where $h_3$ is the gluing used in the standard genus 2 Heegaard decomposition of $S^3$ and $h_1 = h_2 \tau_{c_3} \tau_{c_2} \tau_{c_1}$ with $\tau_{c_i}$ is a right-handed Dehn twist about the curve $c_i$ (see below). Find a surgery description of $H \cup_{h_2} H'$. 

![Diagram of a link exterior with labels c1 and c3]
Exercise 6: Prove that when \( p/q = [x_1, \ldots, x_n] \), \( L(p,q) \) has the following surgery description.