Exercise Set #2

Exercise 1: Compute $\pi_1(L(p,q))$.

Exercise 2: Show that $L(p,q) \cong L(p,q')$ if $qq' \equiv 1 (\text{mod } p)$.

Exercise 3: Find a Heegaard diagram of $\mathbb{RP}^3$.

Exercise 4: What manifold is described by the following Heegaard diagram?

Exercise 5: Let $F$ be a closed orientable surface of genus $g$, and $\alpha_1, \ldots, \alpha_g$ a collection of disjoint simple closed curves on $F$. Prove that the homology classes of $[\alpha_1], \ldots, [\alpha_g] \in H_1(F)$ are linearly independent if and only if the open surface $F \setminus \{\alpha_1, \ldots, \alpha_g\}$ is connected.

Exercise 6: Compute $\pi_1$ of the “random example” pictured below.

Exercise 7: For each positive integer $g$, find a 3-manifold of Heegaard genus $g$. 