

Dehn Surgery and 3-Manifolds Exercise Set #1

Exercise 1: Show that if $K = T_{p,q}$ then M is homeomorphic to $D^2(|p|, |q|)$

Exercise 2: Let K be a knot in S^3 . Show that $H_1(M_K) \cong \mathbb{Z}$.

Exercise 3: Let K be a knot in S^3 . Show that the following are equivalent.

1. K is the unknot;
2. $M_K \cong S^1 \times D^2$;
3. M_K is ∂ -reducible;
4. $\pi_1(M_K) \cong \mathbb{Z}$.

Exercise 4: Let M_1, M_2 be 3-manifolds, with $F_i \subset \partial M_i, i = 1, 2$. Let $h : F_1 \rightarrow F_2$ be a homeomorphism and define $M = M_1 \cup_h M_2$. Show that if each M_i is irreducible and each F_i is incompressible then M is irreducible.

Exercise 5: Let K_1 and K_2 be non-trivial knots. Show that their connected sum $K = K_1 \# K_2$ is a satellite of $K_i, i = 1, 2$.