MATH 4910/5010 Exercises

Chapter 1

Exercise 1: Consider the metric space $(\mathbb{R}^2, d_s)$. For each of the following subsets of $\mathbb{R}^2$, find the closure, interior, and boundary of the subset. Then, determine if the subset is open of closed in $\mathbb{R}^2$.

1. $A = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 < 4\}$
2. $B = \{(x, y) \in \mathbb{R}^2 | y = 2x - 1\}$
3. $C = \{(x, y) \in \mathbb{R}^2 | y < 1 - |x| \text{ and } y \geq 0\}$
4. $D = \{(x, y) \in \mathbb{R}^2 | x, y \in \mathbb{Q}\}$ where $\mathbb{Q}$ is the set of rational numbers.
5. $E = \{(x, y) \in \mathbb{R}^2 | x, y \notin \mathbb{Z}\}$ where $\mathbb{Z}$ is the set of integers.

Exercise 2: Consider $(\mathbb{R}^1, d_s)$. For each positive integer n, let Q_n be the open interval $(-\frac{1}{n}, \frac{1}{n})$. Is each Q_n open in $\mathbb{R}^1$? Is the intersection of all the intervals Q_n open in $\mathbb{R}^1$?

Exercise 3: Consider $\mathbb{Q}$ as a subspace of $\mathbb{R}$. Give an example of a subset which is both open and closed in $\mathbb{Q}$ which isn't the empty set or the total space $\mathbb{Q}$.

Exercise 4: Complete exercise 2 on page 24.

Exercise 5: For each pair of spaces give an explicit homeomorphism between them.

1. $(0, 1)$ and $S^1 - \{(1, 0)\}$
2. $\mathbb{R}^2$ and $\mathbb{B}^2$

Exercise 6: Complete exercise 7 on page 24.

Exercise 7: Complete exercise 10a on page 25.

Chapter 2

Exercise 8: Write the set notation of the abstract simplicial complex realized by the geometric complex in Figure 2.9b on page 46.

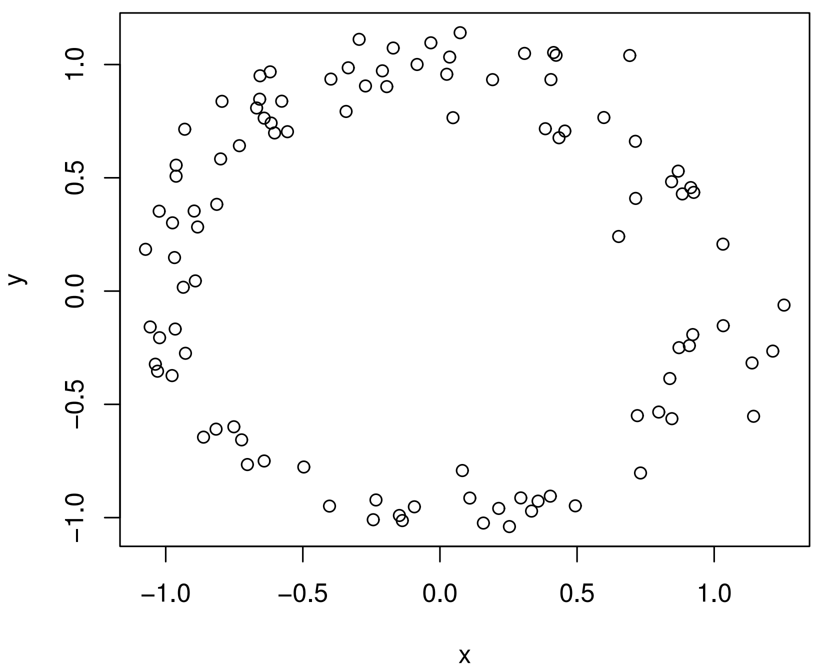
Exercise 9: Let P be the set of the following points in $\mathbb{R}^2$.

$$A = (0,0) \quad B = (2,-2) \quad C = (3,4) \quad D = (4,0) \quad E = (8,2)$$

1. Compute and draw the Čech complex $\mathbb{C}^{2.5}(P)$ and the Vietoris-Rips complex $\mathbb{VR}^{2.5}(P)$.
2. How many triangles are there in $\mathbb{VR}^{3.5}(P)$?

Exercise 10: Give an example of a simplicial complex with vertices in a finite subset $L \subset P$ where a simplex is weakly witnessed but some of its faces are not weakly witnessed.

Exercise 11: Let $\delta = 0.5$. Color in some of the points of a δ-sparse δ-sample in the picture below.



Exercise 12: Complete exercise 3 on page 58.

Exercise 13: Complete exercise 4 on page 58.

Exercise 14: Complete exercise 5 on page 58.

Exercise 15: Complete exercise 10 on page 59.