MATH 4910/5010
Topological Data Analysis

INTRODUCTION

Data Analysis

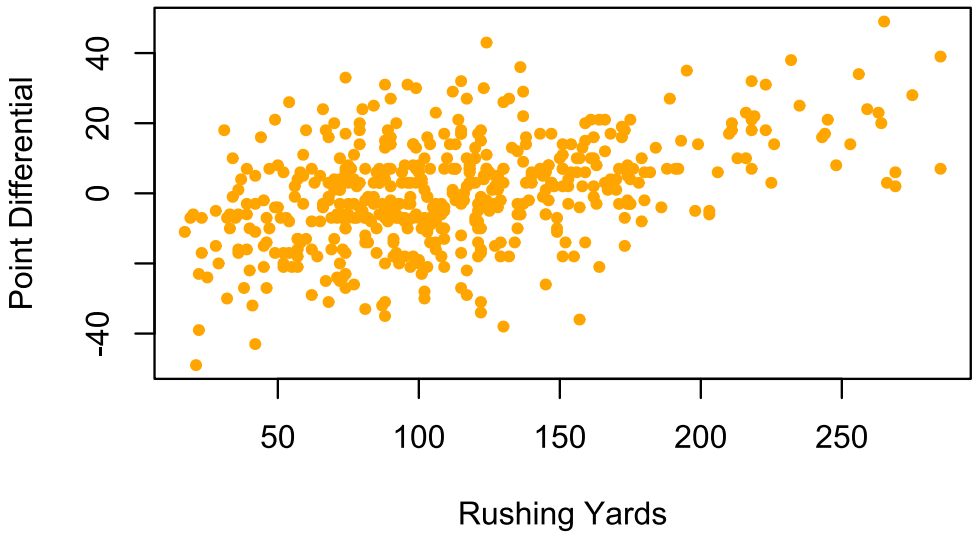
Data

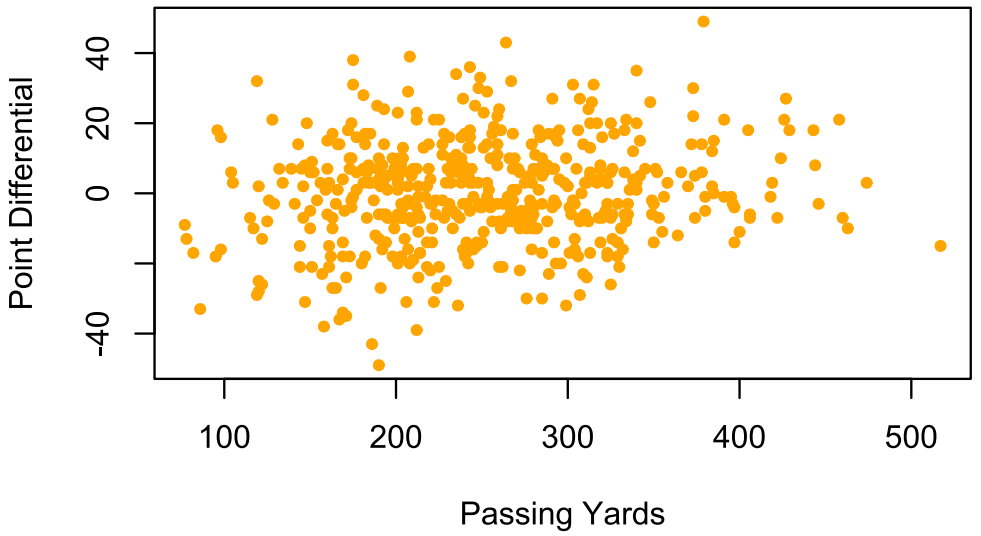
2019 NFL Game Stats

HomeTeam	AwayTeam	Total	H-RushAtt	H-RushYards	H-PassYards	H-Turnover	H-Score	A-RushAtt	A-RushYards	A-PassYards	A-Turnover	A-Score
ARI	DET	45.5	23	112	308	1	27	32	116	385	2	27
CAR	LAR	49.5	23	127	239	3	27	32	166	186	1	30
CHI	GNB	47	15	46	228	1	3	22	47	203	0	10
CLE	TEN	44	20	102	285	3	13	28	123	248	0	43
DAL	NYG	44	30	89	405	0	35	17	151	323	2	17
JAX	KAN	49	16	81	350	2	26	26	113	378	0	40
LAC	IND	44.5	21	125	333	2	30	33	203	190	0	24
MIA	BAL	41	12	21	190	3	10	46	265	379	0	59
MIN	ATL	47	38	172	98	0	28	17	73	304	3	12
NOR	HOU	52	21	148	370	1	30	23	180	268	1	28
NWE	PIT	49	29	99	373	0	33	13	32	276	1	3
NYJ	BUF	41	21	68	175	1	16	25	128	254	4	17
OAK	DEN	42.5	28	98	259	0	24	23	95	268	0	16
PHI	WAS	44	31	123	313	0	32	13	28	380	0	27
SEA	CIN	44	25	72	195	1	21	14	34	418	3	20
TAM	SFO	51	26	121	194	4	17	32	98	166	2	31
ATL	PHI	53	17	57	320	3	24	21	49	255	3	20

Data Analysis

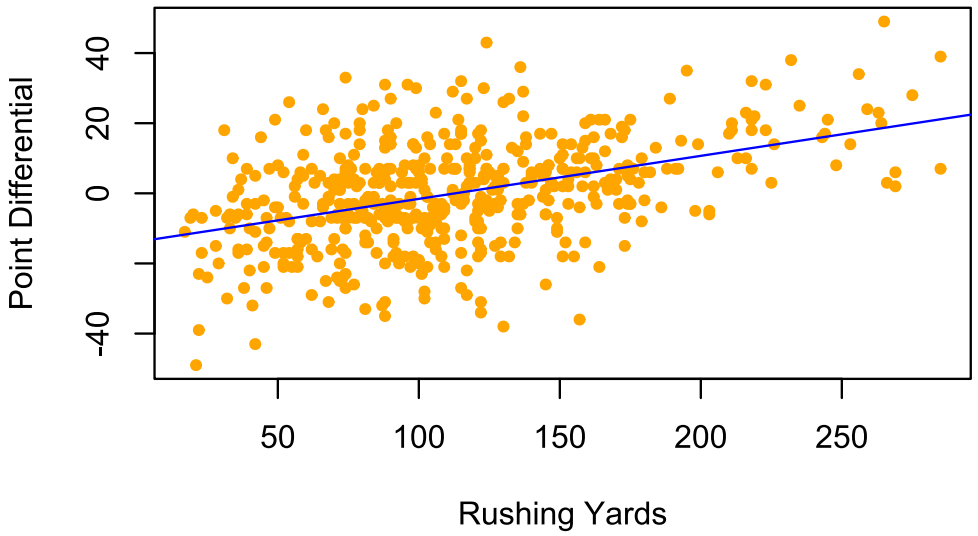
Visualizing Data

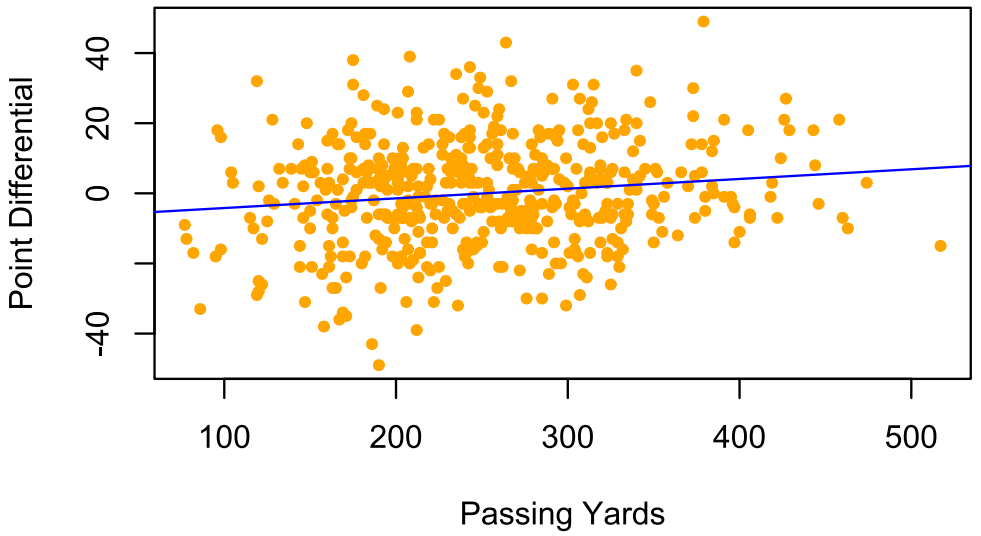




Data Analysis

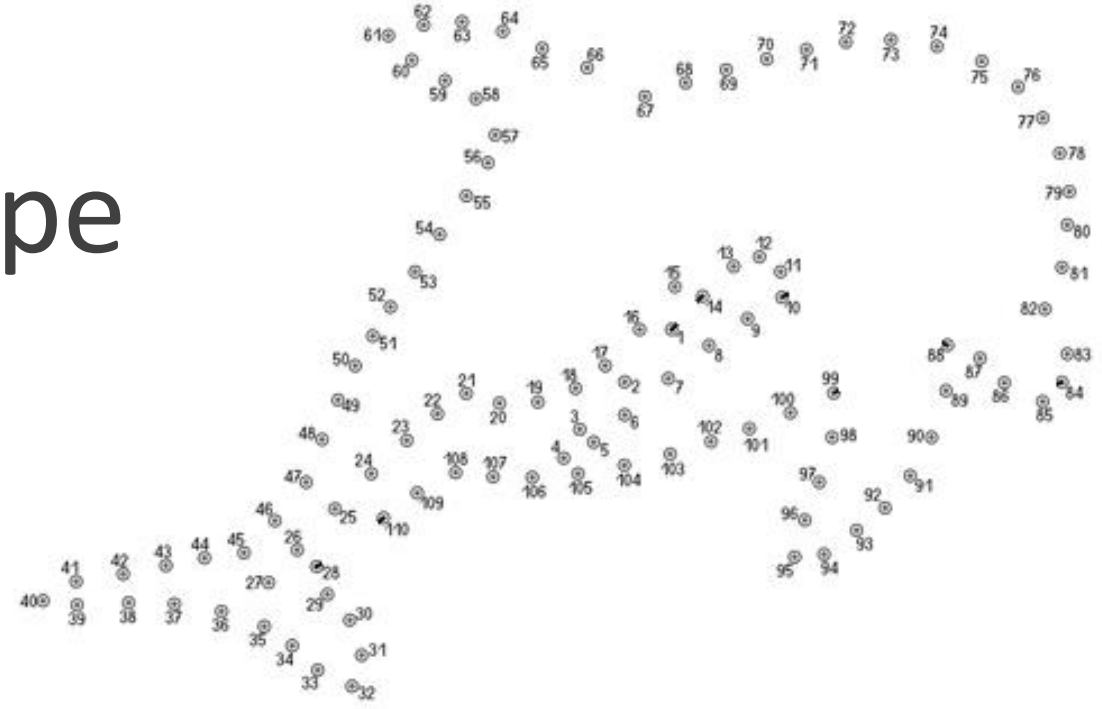
Data Analysis



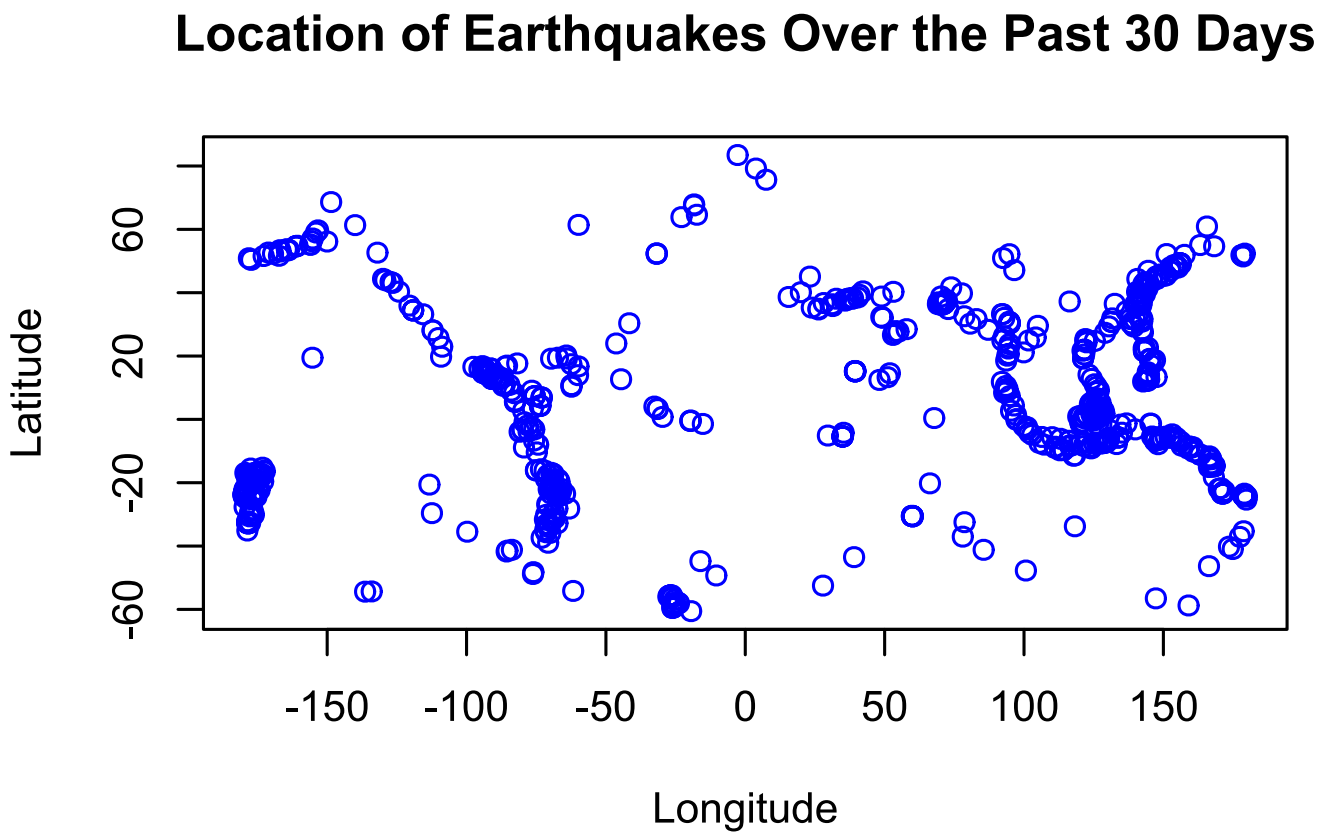


The Shape of Data

Data can have shape and structure!

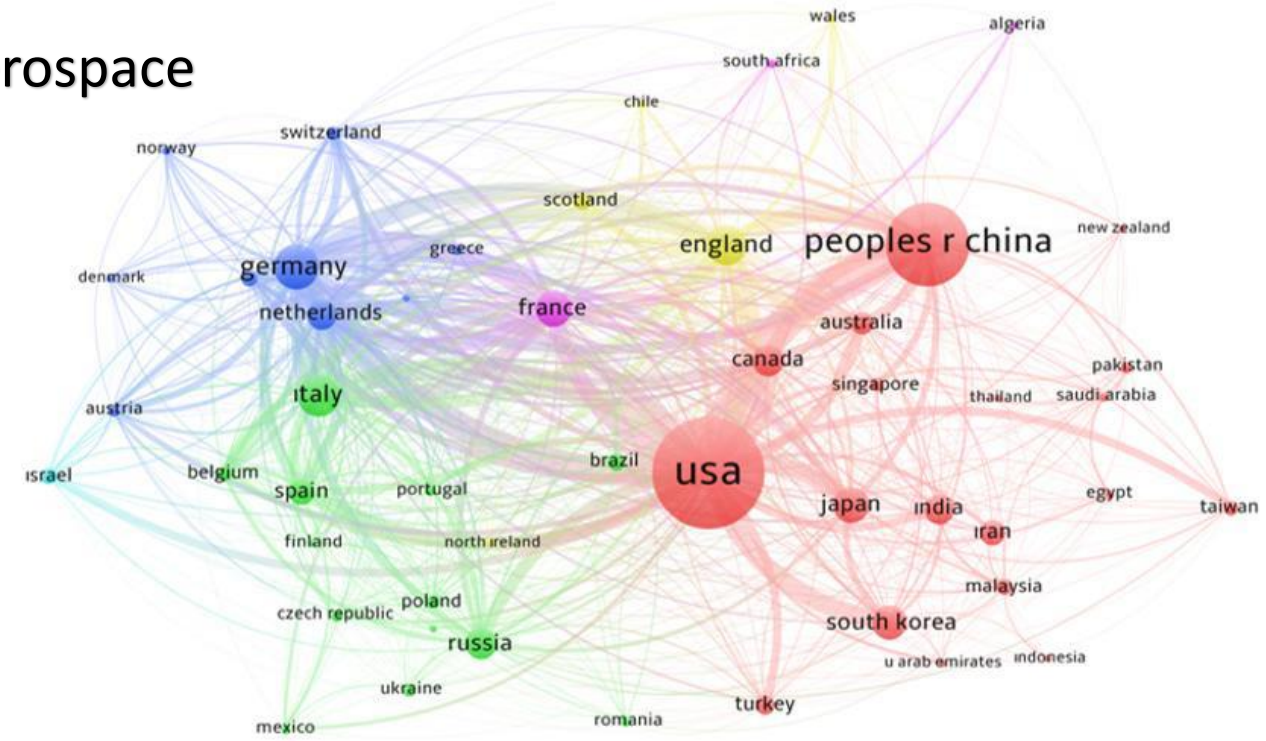


The Shape of Data



The Shape of Data

Collaboration in aerospace engineering



Courtesy of Arnaldo Valdés et al, *Flight Path 2050 and ACARE Goals for Maintaining and Extending Industrial Leadership in Aviation: A Map of the Aviation Technology Space*

Topological Data Analysis

Issues

- What if patterns are hard to find?
- What about high dimensional data?

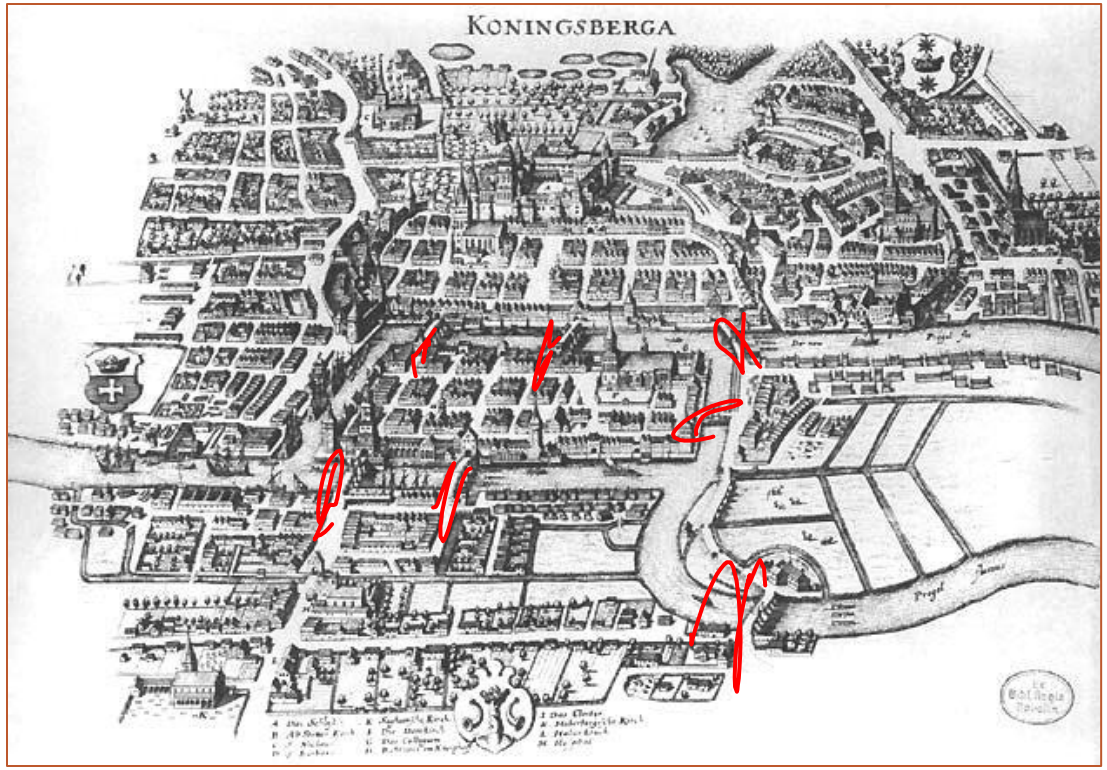
Topological Data Analysis

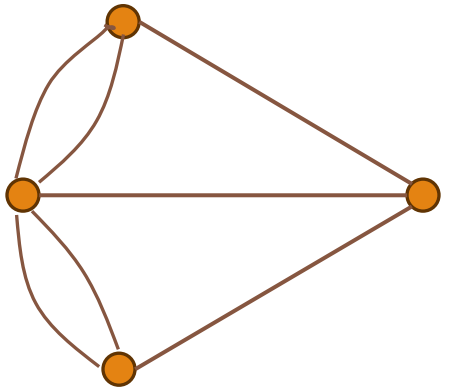
Goal

- Develop effective algorithms for data analysis through a topological lens.

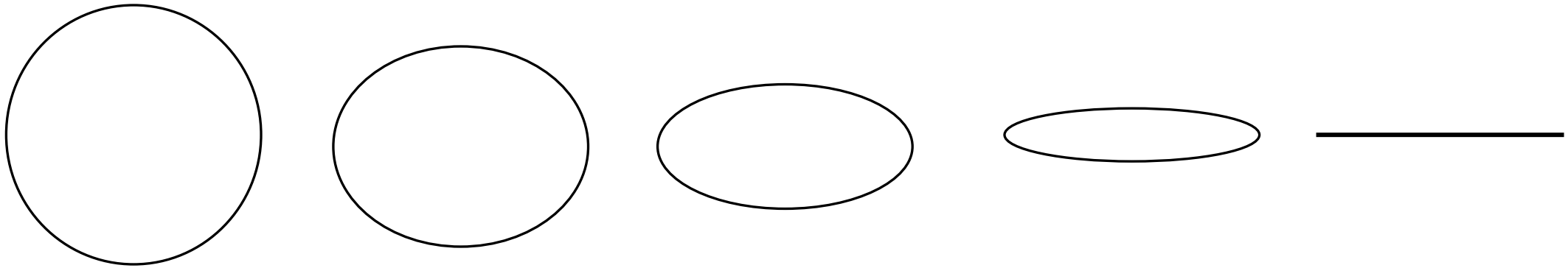
History

Seven Bridges of Königsberg

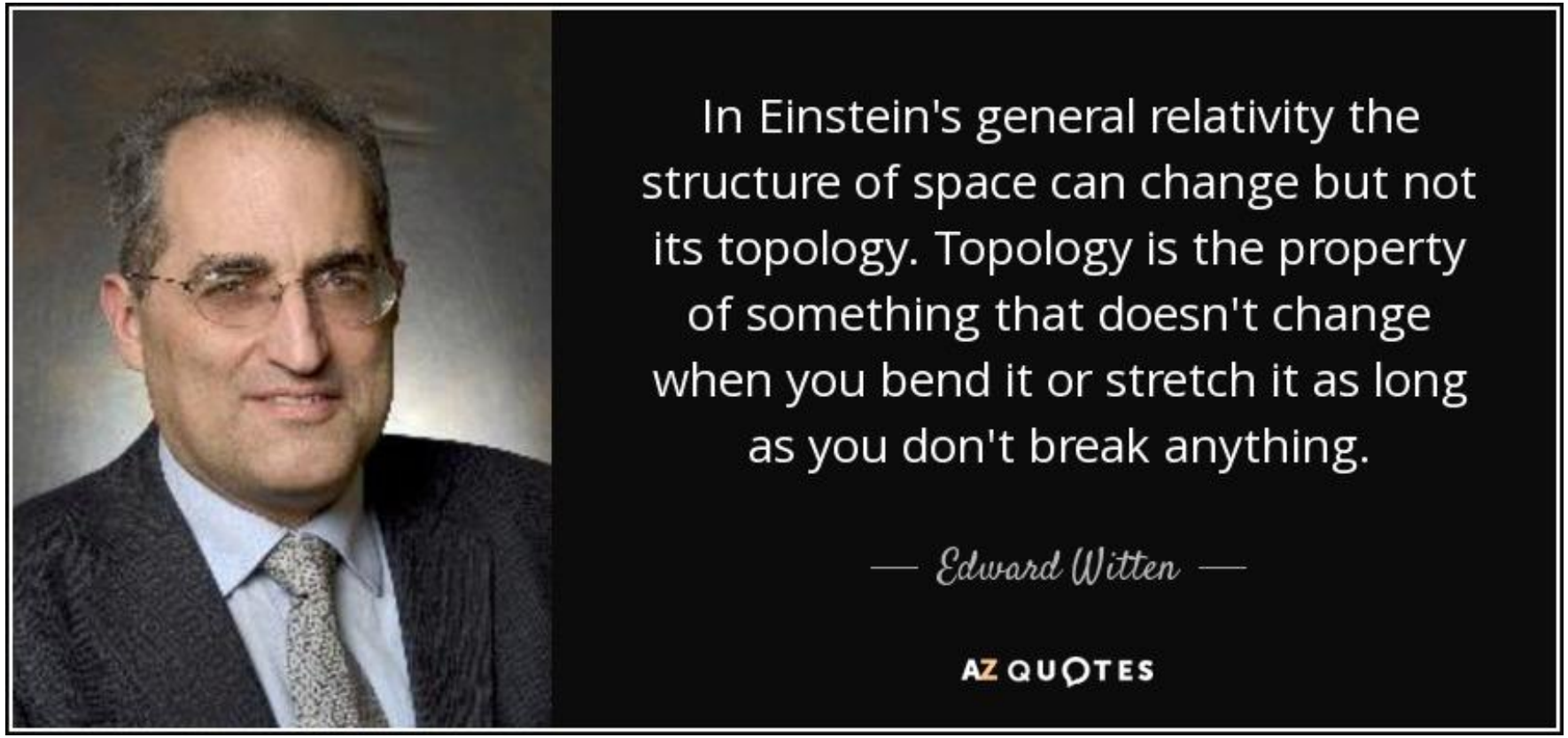




What is Topology?



What is Topology?

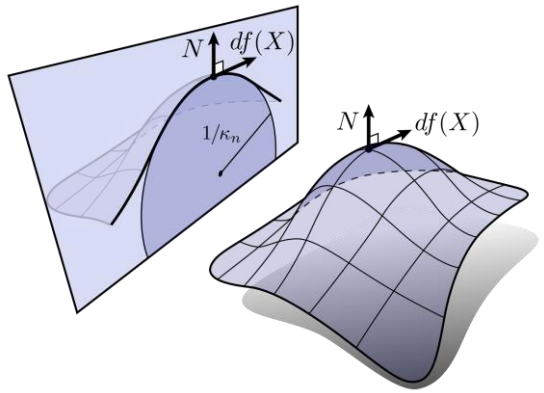


What is Topology?

Geometry

Defined by distances and angles

- How long/big?
- What shape?
- Curvature?



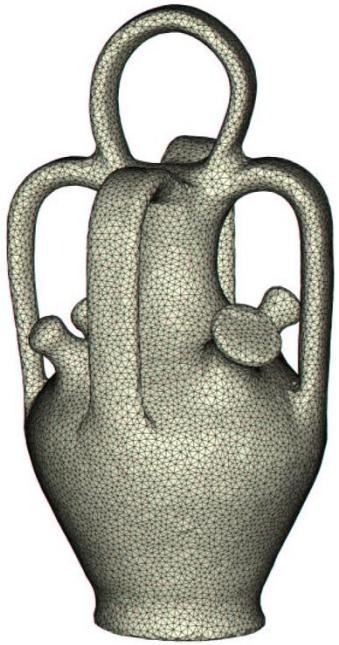
Topology

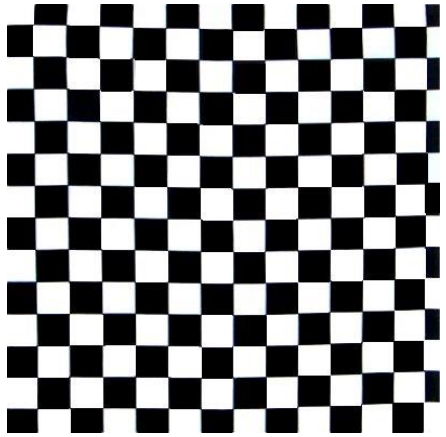
Defined by overall structure of space

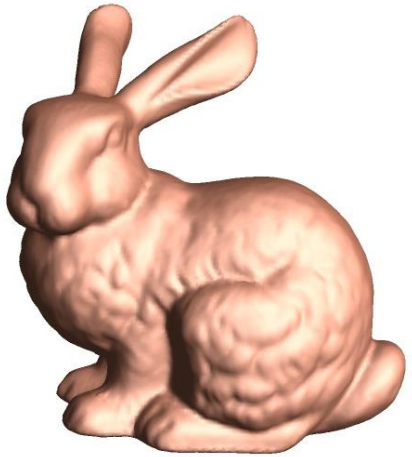
- Is it connected?
- How many pieces?
- Are their “holes”?
- Does it have boundary?

Motivating Examples I

Texture Mapping

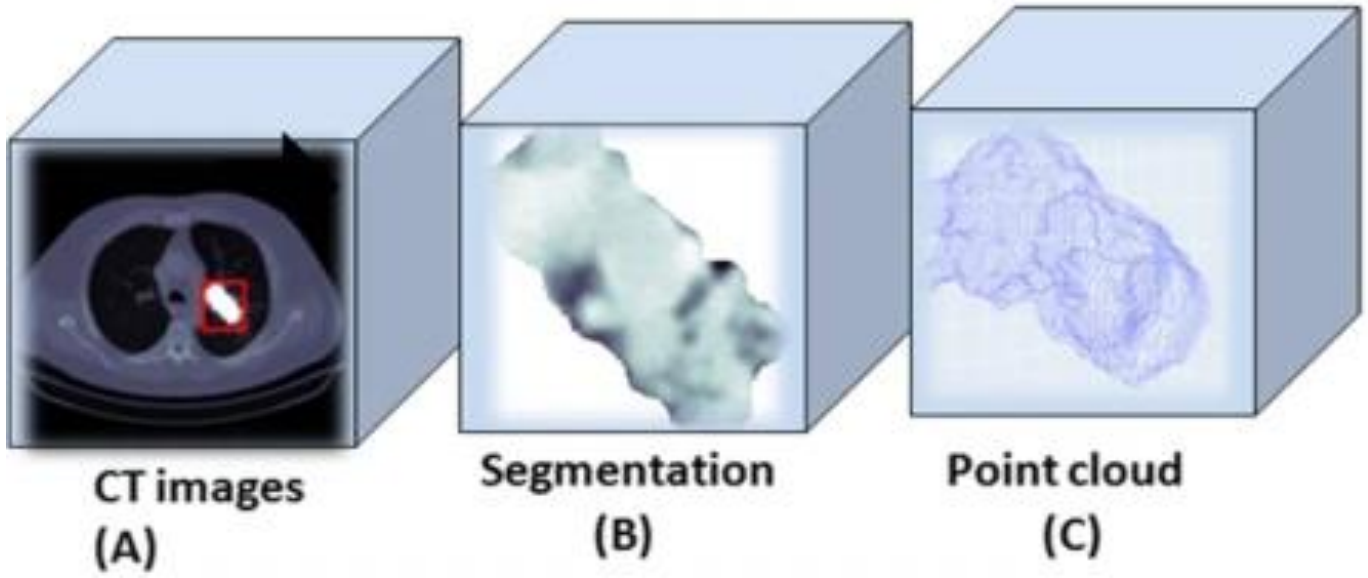






Motivating Examples II

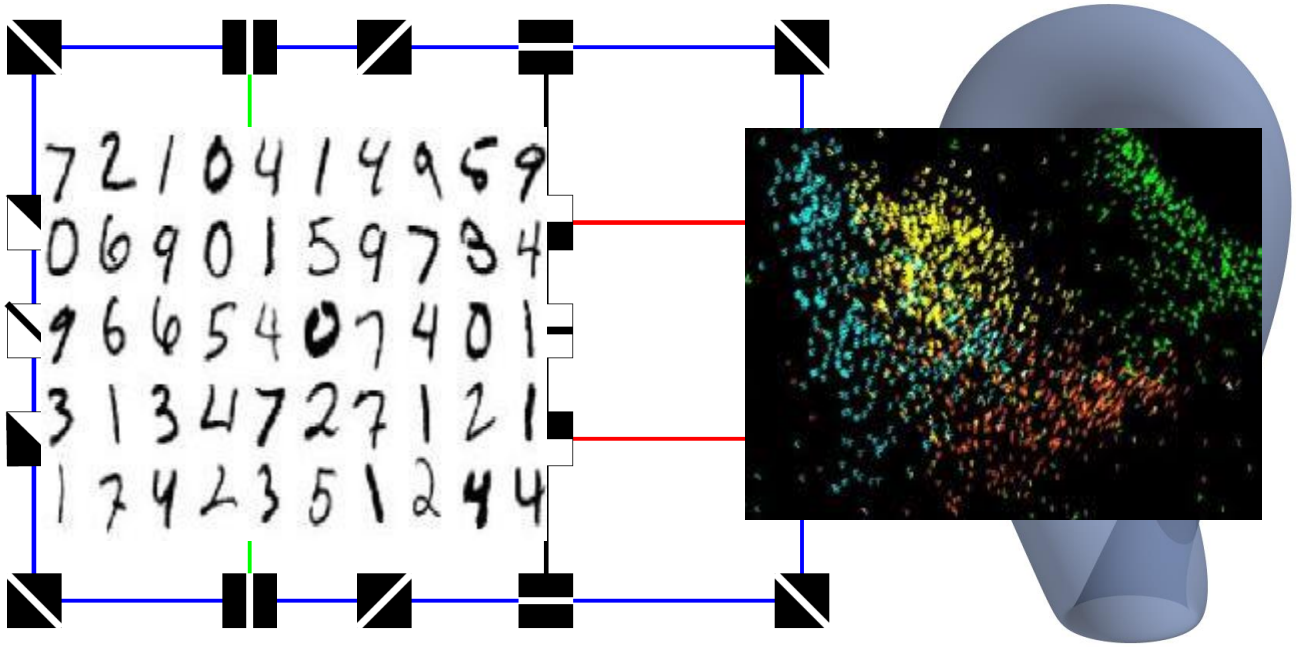
Medical Imaging



Courtesy of Singh et al, *Topological data analysis in medical imaging: current state of the art*

Motivating Examples III

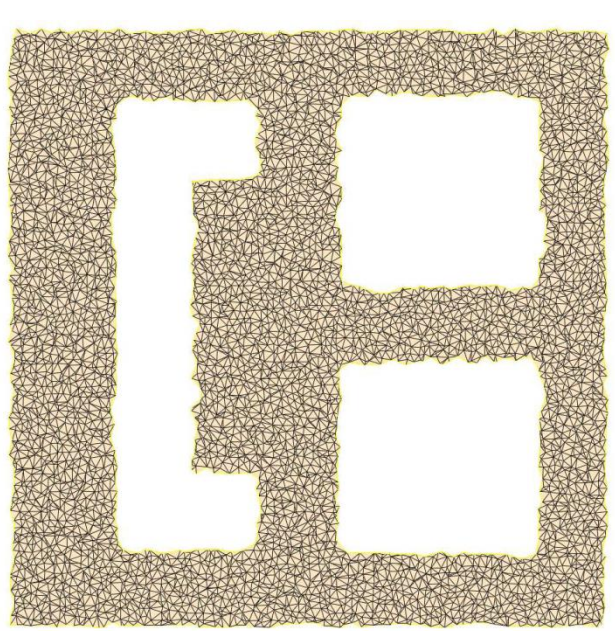
Computer Vision



Courtesy of Carlsson et al, *On the local behavior of spaces of natural images*

Motivating Examples IV

Sensor Networks

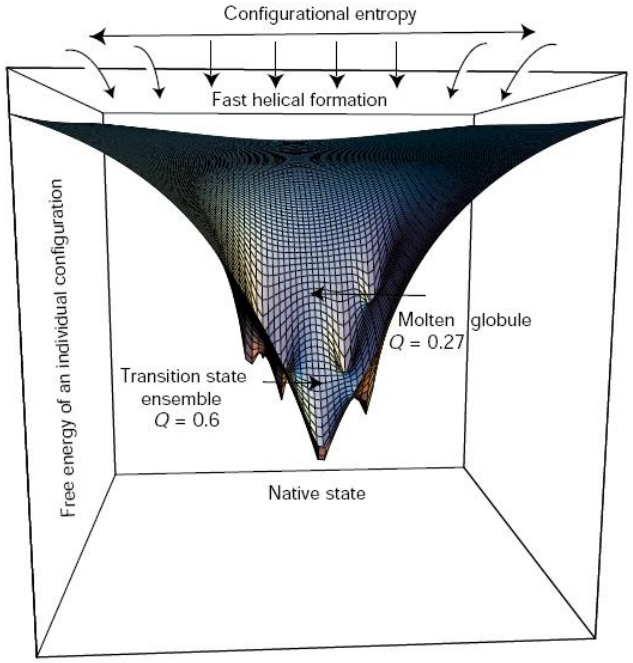


Courtesy of Wang et al. *Boundary recognitio* *ods*

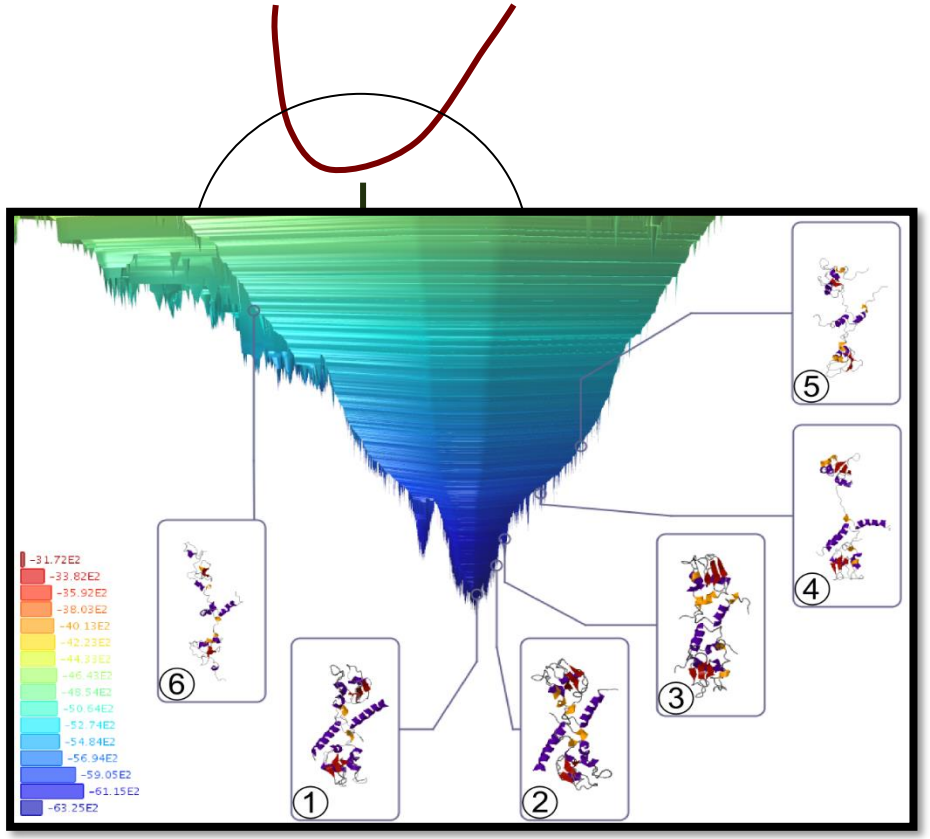
Courtesy of Sarkar et al., Covering space for in-network sensor data storage

Motivating Examples V

Structural Biology

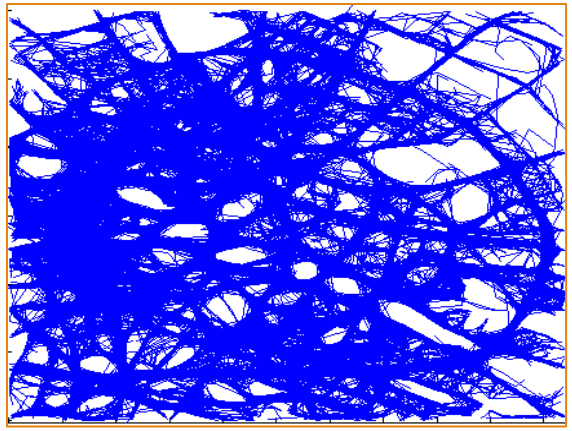


[Wolynes et al., Folding and Design 1996]

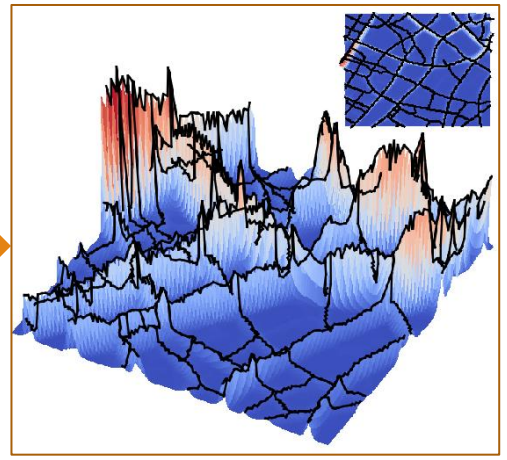


Motivating Examples VI

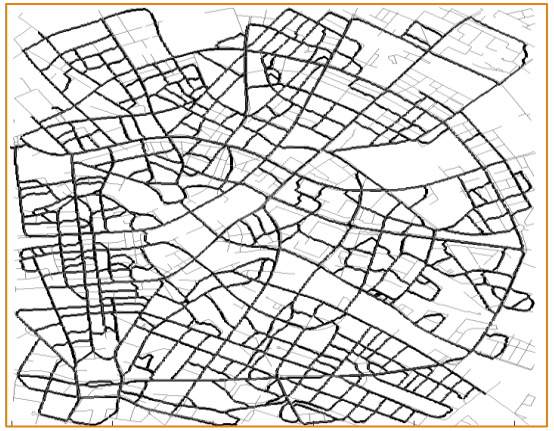
Graph Reconstruction



Input: GPS trajectories



density field + discrete Morse



Output: Road network

MATH 4910/5010

Goals:

- Understand the basics of computational topology
- Introduce persistence homology and related techniques used in topological data analysis(TDA)
- Explore how to use TDA analyze data
- Study applications of these methods to research
- Get hands on experience analyzing data in RStudio

The Plan

1. Introduction to Topology
2. Simplicial (and related) complexes
3. (Simplicial) homology
4. Persistent homology
5. Point cloud data analysis (if time allows)
6. TDA in action

Course Structure

Class activities

- Lectures
- R Demonstrations

Assignments

- Take-home labs using Rstudio (about 4-5)
- First Presentations
  - Exercises using concepts from lecture
  - Around weeks 5-7
- Second Presentations
  - Applications of TDA in research
  - Around weeks 14-15

Text and References

Main Text

- *Computational Topology for Data Analysis,* by T. K. Dey and Y. Wang, Cambridge University Press, 2022.
- *Beginning Data Science in R 4: Data Analysis, Visualization, and Modelling for the Data Scientist* by T. Mailund, Apress Berkeley, CA, 2022. (Available online through Edmon Low Library)

Other references:

- *Computational Topology: An Introduction,* by H. Edelsbrunner and J. Harer, AMS Press, 2009.

Important Links

Couse Website

- https://canvas.okstate.edu
- https://math.okstate.edu/people/jjohnson/courses/2023fall_tda.php

RStudio Links:

- https://cran.rstudio.com
- https://posit.co/downloads