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Stability for a system of the 2D magnetohydrodynamic equations with partial dissipation

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ABSTRACT

This paper examines the stability problem on perturbations near a physically important steady state solution of the 2D magnetohydrodynamic (MHD) system with only partial dissipation. We obtain two main results. The first assesses the asymptotic linear stability with explicit decay rates while the second affirms the global stability in the Sobolev space H^1 setting.

1. Introduction

This paper concerns itself with the following 2D MHD equation with mixed partial dissipation

$$\begin{cases} \partial_t u + u\cdot\nabla u = -\nabla P + \nu\,\partial_{22} u + B\cdot\nabla B, \quad x\in\mathbb{R}^2,\ t>0, \\ \partial_t B + u\cdot\nabla B = B\cdot\nabla u + \eta\partial_{11} B, \quad x\in\mathbb{R}^2,\ t>0, \\ \nabla\cdot u = 0, \quad \nabla\cdot B = 0, \quad x\in\mathbb{R}^2,\ t>0, \end{cases} \tag{1.1}$$

where u represents the fluid velocity, P the pressure and B the magnetic field, and $\nu > 0$, $\eta > 0$ denote the viscosity and the magnetic diffusivity, respectively. We use ∂_1, ∂_2 to denote the derivative in the horizontal and vertical directions, respectively. The incompressible MHD equations model electrically conducting incompressible fluids such as plasmas. In special physical circumstances and under suitable scaling, the partially dissipated system in (1.1) becomes relevant.

This paper focuses on the stability problem on perturbations near the special steady solution (u^0, B^0) given by the background magnetic field

$$u^0(x,t) \equiv (0,0), \quad B^0(x,t) \equiv (1,0). \tag{1.2}$$

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This special equilibrium has physical significance and the stability of (1.2) for the MHD equations was initiated by Alfvén in his work in [1]. The stability problem on the MHD systems has recently gained renewed interests, especially on those systems with partial or no dissipation. Stability and global regularity problems on partially dissipated MHD systems can be extremely challenging. Classical tools designed for fully dissipated systems no longer apply. Due to the efforts of many researchers, mathematically rigorous results have been established for a few MHD systems without full dissipation (see, e.g.,[2–4]). The stability for the ideal MHD system near a background magnetic field is obtained in several beautiful papers [5–8]. The stability for the MHD equations with only velocity dissipation was first established by Lin, Xu and Zhang [9]. This work inspired many further investigations and the stability and precise large time behavior were obtained via many different approaches and in different functional settings (see, e.g.,[10–13]).

To understand the stability problem focused here, we consider the perturbation (u, b) with $b = B - B^{(0)}$. It is easy to check that (u, b) satisfies

$$\begin{cases} \partial_t u + u\cdot\nabla u = -\nabla P + \nu\,\partial_{22} u + b\cdot\nabla b + \partial_1 b, \\ \partial_t b + u\cdot\nabla b = b\cdot\nabla u + \eta\partial_{11} b + \partial_1 u, \\ \nabla\cdot u = \nabla\cdot b = 0, \\ u(x,0) = u_0(x), \quad b(x,0) = b_0(x). \end{cases} \tag{1.3}$$

The stability problem on (1.3) is equivalent to assessing the small data global well-posedness. Due to the lack of the vertical dissipation, this is an extremely difficult problem. To understand the difficulty, we analyze the linearized system

$$\begin{cases} \partial_t u = \nu\,\partial_{22} u + \partial_1 b, \\ \partial_t b = \eta\partial_{11} b + \partial_1 u, \\ \nabla\cdot u = \nabla\cdot b = 0. \end{cases} \tag{1.4}$$

(1.4) can be diagonalized into the following equivalent system

$$\begin{cases} \partial_{tt} u - (\nu\partial_{22} + \eta\partial_{11})\partial_t u + \nu\eta\partial_{1122} u - \partial_{11} u = 0, \\ \partial_{tt} b - (\nu\partial_{22} + \eta\partial_{11})\partial_t b + \nu\eta\partial_{1122} b - \partial_{11} b = 0, \end{cases} \tag{1.5}$$

where u and b each satisfies a generalized damped wave equation. Since (1.4) involves only partial dissipation, showing the linear stability is not trivial. By making suitable assumption on the initial data and introducing a useful technique, we are able to obtain the following asymptotic stability and large-time behavior result.

Theorem 1.1. *Let $s \geq 0$. Assume the initial data $(u_0, b_0) \in H^s(\mathbb{R}^2)$ satisfies $\nabla\cdot u_0 = \nabla\cdot b_0 = 0$. Let (u, b) be the corresponding solution of (1.4) or (1.5).*

(1) *Assume further that $\partial_1 u_0, \partial_1 b_0, \nabla\partial_2 u_0$ and $\nabla\partial_1 b_0$ are all in $H^s(\mathbb{R}^2)$. Then (u, b) satisfies*

$$\|\partial_2 u(t)\|_{H^s} \to 0 \quad \text{and} \quad \|\partial_1 b(t)\|_{H^s} \to 0 \qquad \text{as } t \to \infty. \tag{1.6}$$

(2) *Let $\sigma > 0$. If $\Lambda_1^{-\sigma} u_0, \Lambda_2^{-\sigma} u_0, \Lambda_1^{-\sigma} b_0, \Lambda_2^{-\sigma} b_0 \in H^s(\mathbb{R}^2)$, then (u, b) satisfies*

$$\|u(t)\|_{H^s} + \|b(t)\|_{H^s} \leq C(1+t)^{-\frac{\sigma}{2}}, \tag{1.7}$$

Here the fractional operators $\Lambda_1^\gamma f$ and $\Lambda_2^\gamma f$ with a real number γ are defined via the Fourier transform

$$\widehat{\Lambda_1^\gamma f}(\xi) = |\xi_1|^\gamma\, \widehat{f}(\xi), \quad \widehat{\Lambda_2^\gamma f}(\xi) = |\xi_2|^\gamma\, \widehat{f}(\xi).$$

The first part of Theorem 1.1 establishes the asymptotic stability and the second part gives an explicit decay rate when the initial data is assumed to be in suitable Sobolev spaces of negative indices. When the spatial domain is the whole space, one has to make either the assumption of Sobolev spaces of negative indices or low integrability Lebesgue spaces in order to achieve a decay rate.

We also explore the stability of the full nonlinear system in (1.3) and are able to prove the H^1 stability stated in the following theorem.

Theorem 1.2. *Assume $(u_0, b_0) \in H^1(\mathbb{R}^2)$ satisfies $\nabla \cdot u_0 = \nabla \cdot b_0 = 0$. Then, there exists $\delta > 0$, such that if $\|(u_0, b_0)\|_{H^1} \le \delta$, then the corresponding solution (u, b) of (1.3) satisfies, for a pure constant C and for all $t \ge 0$,*

$$\|(u(t), b(t))\|_{H^1} \le C\delta.$$

To handle the anisotropicity in (1.3), we use in the proof of Theorem 1.2 the anisotropic estimate for triple products.

Lemma 1.3. *Assume that f, g, $\partial_2 g$, h and $\partial_1 h$ are all in $L^2(\mathbb{R}^2)$. Then,*

$$\int\int |fgh| dx \le C\|f\|_{L^2}\|g\|_{L^2}^{\frac{1}{2}}\|\partial_2 g\|_{L^2}^{\frac{1}{2}}\|h\|_{L^2}^{\frac{1}{2}}\|\partial_1 h\|_{L^2}^{\frac{1}{2}}.$$

This lemma can be found in [3]. The rest of this paper is divided into two sections with the first devoted to the proof of Theorem 1.1 and the second to the proof of Theorem 1.2.

2. The asymptotic stability for the linearized system

This section proves Theorem 1.1. We need the following simple fact.

Lemma 2.1. *Assume f satisfies*

$$\int_0^\infty |f(t)|\, dt < \infty, \qquad \int_0^\infty |f'(t)|\, dt < \infty.$$

Then,

$$f(t) \to 0 \quad as\ t \to \infty.$$

Proof of Theorem 1.1. We consider the linearized system (1.4) and its decoupled equivalent in (1.5). (1.5) is obtained by differentiating (1.4) in time and making several substitutions. Dotting the first equation in (1.5) by $\partial_t u$ and integrating over $\mathbb{R}^2$ and also in time lead to

$$\begin{aligned}
&\|\partial_t u(t)\|_{L^2}^2 + \nu\eta\|\partial_{12}u(t)\|_{L^2}^2 + \|\partial_1 u(t)\|_{L^2}^2 \\
&\qquad + 2\nu\int_0^t \|\partial_2\partial_t u(\tau)\|_{L^2}^2 d\tau + 2\eta\int_0^t \|\partial_1\partial_t u(\tau)\|_{L^2}^2 d\tau \\
&= \|\partial_t u_0\|_{L^2}^2 + \nu\eta\|\partial_{12}u_0\|_{L^2}^2 + \|\partial_1 u_0\|_{L^2}^2 \\
&= \|\nu\partial_{22}u_0 + \partial_1 b_0\|_{L^2}^2 + \nu\eta\|\partial_{12}u_0\|_{L^2}^2 + \|\partial_1 u_0\|_{L^2}^2.
\end{aligned} \tag{2.1}$$

Similarly,

$$\begin{aligned}
&\|\partial_t b(t)\|_{L^2}^2 + \nu\eta\|\partial_{12}b(t)\|_{L^2}^2 + \|\partial_1 b(t)\|_{L^2}^2 \\
&\qquad + 2\nu\int_0^t \|\partial_2\partial_t b(\tau)\|_{L^2}^2 d\tau + 2\eta\int_0^t \|\partial_1\partial_t b(\tau)\|_{L^2}^2 d\tau \\
&= \|\eta\partial_{11}b_0 + \partial_1 u_0\|_{L^2}^2 + \nu\eta\|\partial_{12}b_0\|_{L^2}^2 + \|\partial_1 b_0\|_{L^2}^2.
\end{aligned} \tag{2.2}$$

Dotting the first two equations in (1.4) by (u, b) yields

$$\begin{aligned}
&\|u(t)\|_{L^2}^2 + \|b(t)\|_{L^2}^2 + 2\nu\int_0^t \|\partial_2 u(\tau)\|_{L^2}^2 d\tau + 2\eta\int_0^t \|\partial_1 b(\tau)\|_{L^2}^2 d\tau \\
&= \|u_0\|_{L^2}^2 + \|b_0\|_{L^2}^2.
\end{aligned} \tag{2.3}$$

In particular, (2.1), (2.2) and (2.3) imply

$$\int_0^\infty \left(\|\partial_2 u(\tau)\|_{L^2}^2 + \|\partial_t\partial_2 u(\tau)\|_{L^2}^2\right) d\tau, \quad \int_0^\infty \left(\|\partial_1 b(\tau)\|_{L^2}^2 + \|\partial_t\partial_1 b(\tau)\|_{L^2}^2\right) d\tau < \infty$$

Using the simple fact stated in Lemma 2.1, we obtain

$$\|\partial_2 u(t)\|_{L^2} \to 0, \quad \|\partial_1 b(t)\|_{L^2} \to 0.$$

This proves (1.6). Next we prove the decay rate in (1.7). Due to the linearity of the system in (1.4) and (1.5), it suffices to consider the case when $s = 0$. The proof involves a trick. First we have the L^2-estimate,

$$\frac{1}{2}\frac{d}{dt}(\|u\|_{L^2}^2 + \|b\|_{L^2}^2) + \nu\|\partial_2 u\|_{L^2}^2 + \eta\|\partial_1 b\|_{L^2}^2 = 0$$

or

$$\frac{d}{dt}D(t) + G(t) = 0, \tag{2.4}$$

where

$$D(t) := \|u(t)\|_{L^2}^2 + \|b(t)\|_{L^2}^2, \qquad G(t) := 2\nu\|\partial_2 u(t)\|_{L^2}^2 + 2\eta\|\partial_1 b(t)\|_{L^2}^2. \tag{2.5}$$

Applying $\Lambda_1^{-\sigma}$ to the linear system (1.4) and dotting with $(\Lambda_1^{-\sigma}u, \Lambda_1^{-\sigma}b)$ we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}(\|\Lambda_1^{-\sigma}u\|_{L^2}^2 + \|\Lambda_1^{-\sigma}b\|_{L^2}^2) + \nu\|\Lambda_1^{-\sigma}\partial_2 u\|_{L^2}^2 + \eta\|\Lambda_1^{-\sigma}\partial_1 b\|_{L^2}^2\\ &= \int \Lambda_1^{-\sigma}\partial_1 b\cdot\Lambda_1^{-\sigma}u + \Lambda_1^{-\sigma}\partial_1 u\cdot\Lambda_1^{-\sigma}b\,dx = 0.\end{aligned} \tag{2.6}$$

Similarly,

$$\frac{1}{2}\frac{d}{dt}(\|\Lambda_2^{-\sigma}u\|_{L^2}^2 + \|\Lambda_2^{-\sigma}b\|_{L^2}^2) + \nu\|\Lambda_2^{-\sigma}\partial_2 u\|_{L^2}^2 + \eta\|\Lambda_2^{-\sigma}\partial_1 b\|_{L^2}^2 = 0. \tag{2.7}$$

Adding (2.6) and (2.7) yields $\partial_t F(t) \le 0$ or

$$F(t) := \|\Lambda_1^{-\sigma}u\|_{L^2}^2 + \|\Lambda_2^{-\sigma}u\|_{L^2}^2 + \|\Lambda_1^{-\sigma}b\|_{L^2}^2 + \|\Lambda_2^{-\sigma}b\|_{L^2}^2 \le F(0). \tag{2.8}$$

By Plancherel's identity and Hölder's inequality,

$$\begin{aligned}\|u(t)\|_{L^2}^2 &= \int |\widehat{u}(\xi,t)|^2 d\xi = \int (|\xi_2|^2|\widehat{u}(\xi,t)|^2)^{\frac{\sigma}{\sigma+1}}(|\xi_2|^{-2\sigma}|\widehat{u}(\xi,t)|^2)^{\frac{1}{\sigma+1}} d\xi\\ &\le \|\partial_2 u(t)\|_{L^2}^{\frac{2\sigma}{\sigma+1}}\|\Lambda_2^{-\sigma}u(t)\|_{L^2}^{\frac{2}{\sigma+1}}.\end{aligned} \tag{2.9}$$

Similarly,

$$\|b(t)\|_{L^2} \le \|\partial_1 b\|_{L^2}^{\frac{\sigma}{\sigma+1}}\|\Lambda_1^{-\sigma}b(t)\|_{L^2}^{\frac{1}{\sigma+1}}. \tag{2.10}$$

Therefore, by (2.5), (2.8), (2.9) and (2.10),

$$D(t) \le C G(t)^{\frac{\sigma}{1+\sigma}} F(t)^{\frac{1}{1+\sigma}} \le C G(t)^{\frac{\sigma}{1+\sigma}} F(0)^{\frac{1}{1+\sigma}}$$

or

$$G(t) \ge C\, F(0)^{-\frac{1}{\sigma}}\, D(t)^{1+\frac{1}{\sigma}}. \tag{2.11}$$

Inserting (2.11) in (2.4) and then solving the resulting differential inequality yield

$$D(t) \le (C_1(\sigma, \|(u_0,b_0)\|_{L^2}) + C_2(\sigma, A_0)t)^{-\sigma}, \tag{2.12}$$

where C_1 and C_2 are constants and

$$A_0 = \|\Lambda_1^{-\sigma}u_0\|_{L^2}^2 + \|\Lambda_2^{-\sigma}u_0\|_{L^2}^2 + \|\Lambda_1^{-\sigma}b_0\|_{L^2}^2 + \|\Lambda_2^{-\sigma}b_0\|_{L^2}^2.$$

Clearly, (2.12) implies the desired result in (1.7). This completes the proof of Theorem 1.1. □

3. Proof of Theorem 1.2

This section proves Theorem 1.2.

Proof of Theorem 1.2. For $t \geq 0$, define

$$E(t) = \|(u(t), b(t))\|_{H^1}^2 + 2\nu \int_0^t \|\partial_2 u(\tau)\|_{H^1}^2 d\tau + 2\eta \int_0^t \|\partial_1 b(\tau)\|_{H^1}^2 d\tau. \tag{3.1}$$

First, we have the L^2-estimate

$$\|(u(t), b(t))\|_{L^2}^2 + 2\nu \int_0^t \|\partial_2 u(\tau)\|_{L^2}^2 d\tau + 2\eta \int_0^t \|\partial_1 b(\tau)\|_{L^2}^2 d\tau = \|(u_0, b_0)\|_{L^2}^2, \tag{3.2}$$

where we have used integration by parts. To estimate $\|\nabla u\|_{L^2}$ and $\|\nabla b\|_{L^2}$, we make use of the equations of the vorticity $\omega = \nabla \times u$ and the current density $j = \nabla \times b$,

$$\begin{cases} \partial_t \omega + u \cdot \nabla \omega = \nu\, \partial_{22}\omega + b \cdot \nabla j + \partial_1 j, \\ \partial_t j + u \cdot \nabla j = \eta \partial_{11} j + b \cdot \nabla \omega + \partial_1 \omega + Q, \end{cases} \tag{3.3}$$

where $Q := 2\partial_1 b_1(\partial_2 u_1 + \partial_1 u_2) - 2\partial_1 u_1(\partial_2 b_1 + \partial_1 b_2)$. Dotting (3.3) with (ω, j) yields

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}(\|\omega\|_{L^2}^2 + \|j\|_{L^2}^2) + \nu\|\partial_2\omega\|_{L^2}^2 + \eta\|\partial_1 j\|_{L^2}^2 = \int Qj dx \\ &= \int 2\partial_1 b_1 \partial_2 u_1 j dx + \int 2\partial_1 b_1 \partial_1 u_2 j dx - \int 2\partial_1 u_1 \partial_2 b_1 j dx - \int 2\partial_1 u_1 \partial_1 b_2 j dx \\ &:= M_1 + M_2 + M_3 + M_4. \end{aligned} \tag{3.4}$$

By Lemma 1.3,

$$M_1 \leq C\|\partial_1 b_1\|_{L^2}\|\partial_2 u_1\|_{L^2}^{\frac{1}{2}}\|\partial_2\partial_2 u_1\|_{L^2}^{\frac{1}{2}}\|j\|_{L^2}^{\frac{1}{2}}\|\partial_1 j\|_{L^2}^{\frac{1}{2}} \tag{3.5}$$

and

$$M_2 \leq C\|\partial_1 b_1\|_{L^2}\|\partial_1 u_2\|_{L^2}^{\frac{1}{2}}\|\partial_2\partial_1 u_2\|_{L^2}^{\frac{1}{2}}\|j\|_{L^2}^{\frac{1}{2}}\|\partial_1 j\|_{L^2}^{\frac{1}{2}}. \tag{3.6}$$

To bound M_3, we write $j = \partial_1 b_2 - \partial_2 b_1$ and

$$\begin{aligned} M_3 &= -2\int \partial_1 u_1 \partial_2 b_1(\partial_1 b_2 - \partial_2 b_1) dx \\ &= 2\int \partial_2 u_2 \partial_2 b_1 \partial_1 b_2 dx + 4\int u_1 \partial_1 \partial_2 b_1 \partial_2 b_1 dx \\ &\leq C\|\partial_2 u_2\|_{L^2}\|\partial_2 b_1\|_{L^2}^{\frac{1}{2}}\|\partial_1\partial_2 b_1\|_{L^2}^{\frac{1}{2}}\|\partial_1 b_2\|_{L^2}^{\frac{1}{2}}\|\partial_2\partial_1 b_2\|_{L^2}^{\frac{1}{2}} \\ &\quad + C\|\partial_1\partial_2 b_1\|_{L^2}\|u_1\|_{L^2}^{\frac{1}{2}}\|\partial_2 u_1\|_{L^2}^{\frac{1}{2}}\|\partial_2 b_1\|_{L^2}^{\frac{1}{2}}\|\partial_1\partial_2 b_1\|_{L^2}^{\frac{1}{2}}. \end{aligned} \tag{3.7}$$

By Lemma 1.3,

$$M_4 \leq C\|\partial_2 u_2\|_{L^2}\|\partial_1 b_2\|_{L^2}^{\frac{1}{2}}\|\partial_2\partial_1 b_2\|_{L^2}^{\frac{1}{2}}\|j\|_{L^2}^{\frac{1}{2}}\|\partial_1 j\|_{L^2}^{\frac{1}{2}}. \tag{3.8}$$

Inserting (3.5), (3.6), (3.7), (3.8) in (3.4) and using the fact that $\|\omega\|_{L^2} = \|\nabla u\|_{L^2}$ and $\|j\|_{L^2} = \|\nabla b\|_{L^2}$, we have, by combining with (3.2),

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}(\|u\|_{H^1}^2 + \|b\|_{H^1}^2) + \nu\|\partial_2 u\|_{H^1}^2 + \eta\|\partial_1 b\|_{H^1}^2 \\ &\leq C\|u\|_{H^1}^{\frac{1}{2}}\|b\|_{H^1}^{\frac{1}{2}}\|\partial_2 u\|_{H^1}^{\frac{1}{2}}\|\partial_1 b\|_{H^1}^{\frac{3}{2}} + C\|\partial_2 u\|_{H^1}\|\partial_1 b\|_{H^1}\|b\|_{H^1}. \end{aligned} \tag{3.9}$$

Integrating (3.9) in time and recalling (3.1) yield

$$\begin{aligned} E(t) \leq\ & E(0) + C\int_0^t \|u\|_{H^1}^{\frac{1}{2}}\|b\|_{H^1}^{\frac{1}{2}}\|\partial_2 u\|_{H^1}^{\frac{1}{2}}\|\partial_1 b\|_{H^1}^{\frac{3}{2}}d\tau \\ & + C\int_0^t \|\partial_2 u\|_{H^1}\|\partial_1 b\|_{H^1}\|b\|_{H^1}d\tau \\ \leq\ & E(0) + CE(t)^{\frac{3}{2}}, \end{aligned}$$

By a bootstrap argument (see, e.g., [14, p. 20]), there is $\delta > 0$ such that, if $E(0) < \delta$, then

$$E(t) \leq C\delta$$

for a pure constant C and for all $t \geq 0$. This proves Theorem 1.2. □

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