1. Let \([a_n, b_n]\) denote the successive intervals that arise from applying the bisection method to find a root of a continuous function \(f(x)\). Let \(c_n = \frac{1}{2}(a_n + b_n)\), \(r = \lim_{n \to \infty} c_n\), and \(\epsilon_n = r - c_n\).

(a) Show that \(|\epsilon_n| \leq 2^{-n} |b - a|\).

(b) Show that \(\epsilon_n = O(2^{-n})\) as \(n \to \infty\).

2. Using the bisection algorithm, find a root of \(f(x) = x - \tan(x)\) in the interval \([1, 2]\).

3. Using the bisection algorithm, find a root of \(f(x) = 2^{-x} + e^x + 2\cos(x) - 6\) on \([1, 3]\).

4. With a hand held calculator, perform four iterations of Newton’s Method to find an approximate zero of \(f(x) = 4x^3 - 2x^2 + 3\) starting with \(x_0 = -1\).

5. Write a computer program to solve \(x = \tan x\) by means of Newton’s method. Find the roots nearest 4.5 and 7.7.

6. Write a computer program to solve \(x^3 + 3x = 5x^2 + 7\) by Newton’s Method. Take ten steps starting with \(x_0 = 5\).