# Math 4263

Homework Set 3

1. Apply Separation of Variables to the Wave Equation $\phi_{tt} - c^2\phi_{xx} = 0$ to obtain four distinct one-parameter families of linearly independent solutions. Then, construct a solution to

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0 \quad , \quad 0 \leq x \leq 2 \quad , \quad 0 \leq t \\ u(0,t) &= 0 \quad , \quad 0 \leq t \\ u(2,t) &= 0 \quad , \quad 0 \leq t \\ u(x,0) &= \sin(3\pi x) \quad , \quad 0 \leq x \leq 2 \\ u_t(x,0) &= \sin(\pi x) \quad , \quad 0 \leq x \leq 2 \end{aligned}$$

2. Solve

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0 \quad , \quad -\infty \leq x < \infty \\ u(x,0) &= f(x) \quad , \quad -\infty \leq x < \infty \\ u_t(x,0) &= g(x) \quad , \quad -\infty \leq x < \infty \end{aligned}$$

by making a change of variables $u(x,t) = x + ct$, $v(x,t) = x - ct$.

3. Show that the Laplace operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

under the change of coordinates

$$\begin{array}{c} x = r\cos\theta \\ y = r\sin\theta \end{array} \quad \longleftrightarrow \quad \begin{array}{c} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{array}$$

becomes

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^1}\frac{\partial^2}{\partial \theta^2}$$

4. Construct a solution to Laplace's equation on the unit disc $D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\right\}$ satisfying $\phi(\cos(\theta), \sin(\theta)) = 1 + 3\sin(\theta)$, $0 \leq \theta < 2\pi$.

5. Construct a solution to Laplace's equation on the annular region

$$R = \left\{(x,y) \in \mathbb{R}^2 \mid a^2 \leq x^2 + y^2 \leq b^2\right\}$$

subject to the boundary conditions

$$\begin{aligned} u(a\cos(\theta), a\sin(\theta)) &= g(\theta) \\ u(b\cos(\theta), b\sin(\theta)) &= h(\theta) \end{aligned}$$