Math 4263
Homework Set 1

1. Solve the following PDE/BVP

$$\begin{aligned} 2u_t + 3u_x &= 0 \\ u(x,0) &= \sin(x) \end{aligned}$$

2. Solve the following PDE/BVP

$$\begin{aligned} u_x + e^x u_y &= 0 \\ u(0,y) &= y^2 \end{aligned}$$

3.

(a) Find the curves $\boldsymbol{\gamma} : t \longrightarrow (x(t), y(t))$ such that

$$\frac{dx}{dt} = x \qquad , \qquad \frac{dy}{dt} = y$$

that cross the line $y = 1$ at $t = 0$.

(b) Solve the following PDE/BVP

$$\begin{aligned} x\phi_x + y\phi_y &= y \\ \phi(x,1) &= x+1 \end{aligned}$$

by first finding solutions of the PDE/BVP along the curves $\boldsymbol{\gamma}$ determined in Part (a) and then extending these solutions coherently to arbitrary points in $\mathbb{R}^2$.

4.

(a) Find curves $\gamma : t \to [x(t), y(t)]$ such that

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= 2y \end{aligned}$$

(b) Solve the following PDE/BVP:

$$\begin{aligned} y\frac{\partial \phi}{\partial x} + 2y\frac{\partial \phi}{\partial y} &= xy \\ \phi(x,1) &= x+2 \end{aligned}$$

5. Use the Method of Characteristics to show that the solution of

$$uu_x + u_y = 0 \qquad , \qquad u(x,0) = f(x)$$

is given implicitly by

$$u = f(x - uy)$$

and verify this result by direct differentiation.

6. Use the Method of Characteristics to solve

$$\begin{aligned} \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \phi &= e^{x+2y} \\ \phi(x,0) &= x \end{aligned}$$