Math 4233
Homework Set 5

1. For each of the following PDEs, try using the method of separation of variables to replace the PDE by a pair of ODEs.

   (a) \( xu_{xx} + u_t = 0 \)

   (b) \( u_{xx} + u_{xt} + u_t = 0 \)

   (c) \( tu_{xx} + xu_t = 0 \)

   (d) \( [p(x)u_x]_x - r(x)u_{tt} = 0 \)

   (e) \( u_{xx} + u_{yy} + xu = 0 \)

2. Find the solution of the following heat conduction problem:

   \( 4u_t - u_{xx} = 0, \quad 0 < x < 2, \quad t > 0 \)

   \( u(0, t) = 0 \)

   \( u(2, t) = 0 \)

   \( u(x, 0) = 2 \sin \left( \frac{\pi x}{2} \right) - \sin(\pi x) + 4 \sin(2\pi x) \)

3. Find the solution of:

   \( 4u_t - u_{xx} = 0, \quad 0 < x < 2, \quad t > 0 \)

   \( u(0, t) = 2 \)

   \( u(2, t) = -2 \)

   \( u(x, 0) = 2 \sin(\pi x) \)

4. Show that the wave equation

   \( (*) \quad u_{tt} - a^2 u_{xx} = 0 \)

   can be reduced to the form

   \( u_{\xi\eta} = 0 \)

   by a change for variables \(\xi = x - at, \eta = x + at\). Conclude that the any solution of \( (*) \) can be written as

   \( u(x, t) = \phi(x - at) + \psi(x + at) \).

5. Find the solution of Laplace’s equation

   \( u_{xx} + u_{yy} = 0 \)

   satisfying the boundary conditions

   \( u(x, 0) = 0 \), \( u(x, b) = g(x) \)

   \( u(0, y) = 0 \), \( u(a, y) = 0 \)

6. Express the 2-dimensional Laplace equation

   \( u_{xx} + u_{yy} = 0 \)

   in terms of polar coordinates \((r, \theta)\) and use separation of variables to reduce it to the solution of a pair of ordinary differential equations.