MATH 4063-5023
Homework Set 5

1. Suppose that V is a finitely generated vector space and $\phi : V \to W$ is a linear transformation. Show that $Range(\phi) \subset W$ is finitely generated.

2. Suppose S is a subspace of a finitely generated vector space V. Show that V/S is finitely generated.

3. Suppose S is a subspaces of a finitely generated vector space V. Find a basis for V/S.

4. Suppose S is a subspace of a finitely generated vector space V, show that $\dim(V) = \dim(S) + \dim(V/S)$.

5. Suppose $\phi : V \to W$ is a linear transformation between two finite-dimensional vectors spaces. Show that $Range(\phi)$ is isomorphic to $V/\ker(\phi)$. (Hint: two finite-dimensional vector spaces (over a common field $\mathbb{F}$) are isomorphic if and only if they have the same dimension.