Section 6.1

6.1.1 Let \( S^* = (0,1] \times [0,2\pi) \) and defined \( T(r, \theta) = (r \cos(\theta), r \sin(\theta)) \). Determine the image set \( S \) and show that \( T \) is one-to-one on \( S^* \).

6.1.2 Let \( D^* = [0,1] \times [0,1] \) and define \( T \) on \( D^* \) by \( T(u,v) = (-u^2 + 4u,v) \). Find \( D \). Is \( T \) one-to-one?

6.1.3 Let \( D^* = [0,1] \times [0,1] \) and define \( T \) on \( D^* \) by \( T(u,v) = (uv,u) \). Find \( D \). Is \( T \) one-to-one? If not, can we eliminate some subset of \( D^* \) so that on the remainder \( T \) is one-to-one?

6.1.4 Let \( T(x) = Ax \) where \( A \) is a \( 2 \times 2 \) matrix. Show that \( T \) is one-to-one if and only if the determinant of \( A \) is non-zero.

6.1.5 Suppose \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) is linear; i.e., \( T(x) = Ax \), where \( A \) is a \( 2 \times 2 \) matrix. Show that if \( \det A \neq 0 \), then \( T \) takes parallelograms to parallelograms. (Hint: any parallelogram in \( \mathbb{R}^2 \) can be described as a set \( \{ \mathbf{r} = \mathbf{p} + \lambda \mathbf{v} + \mu \mathbf{w} \mid \lambda,\mu \in [0,1] \} \) where \( \mathbf{p}, \mathbf{v}, \mathbf{w} \) are suitable vectors in \( \mathbb{R}^2 \) with \( \mathbf{v} \) not a scalar multiple of \( \mathbf{w} \).

Section 6.2

6.2.1 Let \( D \) be the unit circle. Evaluate
\[
\int_D \exp \left( x^2 + y^2 \right) \, dx \, dy
\]
by making a change of variables to polar coordinates.

6.2.2 Let \( D \) be the region \( 0 \leq y \leq x \) and \( 0 \leq x \leq 1 \). Evaluate
\[
\int_D (x+y) \, dx \, dy
\]
by making the change of variables \( x = u + v, \ y = u - v \). Check your answer by evaluating the integral directly by using an iterated integral.

6.2.3 Let \( T(u,v) = (x(u,v),y(u,v)) \) be the mapping defined by \( T(u,v) = (4u,2u+3v) \). Let \( D' \) be the region in \( u - v \) plane corresponding to the rectangle \([0,1] \times [1,2] \). Find \( D = T(D') \) and evaluate
(a) \( \int_D xy \, dA \)
(b) \( \int_D (x-y) \, dA \)

6.2.4 Define \( T(u,v) = (u^2 - v^2, 2uv) \). Let \( D' \) be the set of \( (u,v) \) with \( u^2 + v^2 \leq 1, \ u \geq 0, \ v \geq 0 \). Find \( T(D') \) = \( D \). Evaluate
\[
\int_D dA
\]

6.2.5 Let \( T(u,v) \) be as in Exercise 6.2.4. By making this change of variables evaluate
\[
\int_D \frac{dA}{\sqrt{x^2 + y^2}}
\]

6.2.6 Integrate \( xe^{x^2 + y^2} \) over the cylinder \( x^2 + y^2 \leq 4, -2 \leq z \leq 3. \)