

Math 2233  
Homework Set 8

1. Determine the lower bound for the radius of convergence of series solutions about each given point  $x_0$ .

- (a)  $y'' + 4y' + 6xy = 0$  ,  $x_0 = 0$
- (b)  $(x - 1)y'' + xy' + 6xy = 0$  ,  $x_0 = 4$
- (c)  $(4 + x^2)y'' + 4xy' + y = 0$  ,  $x_0 = 0$
- (d)  $(1 + x^2)y'' + 4xy' + y = 0$  ,  $x_0 = 2$

2. Determine the singular points of the following differential equations and state whether they are regular or irregular singular points.

- (a)  $xy'' + (1 - x)y' + xy = 0$
- (b)  $x^2(1 - x)^2y'' + 2xy + 4y = 0$
- (c)  $(1 - x^2)^2y'' + x(1 - x)y' + (1 + x)y = 0$

3. The following differential equation has a regular singular point at  $x = 0$ . Determine the indicial equations, the roots of the indicial equations, the recursion relations, and the first four terms of two linearly independent series solutions.

$$2xy'' + y' + xy = 0 \quad .$$

4. The following differential equation has a regular singular point at  $x = 0$ . Determine the indicial equation and the recursion relations corresponding to the largest root of the indicial equation. Write down the first four terms of the corresponding solution.

- (a)  $xy'' + y = 0$
- (b)  $xy'' + (1 - x)y' - y = 0$