Math 2233
Homework Set 7

1. Determine the lower bound for the radius of convergence of series solutions about each given point x_o.

(a) $y'' + 4y' + 6xy = 0$, $x_0 = 0$
(b) $(x-1)y'' + xy' + 6xy = 0$, $x_0 = 4$
(c) $\left(4 + x^2\right) y'' + 4xy' + y = 0$, $x_0 = 0$
(d) $\left(1 + x^2\right) y'' + 4xy' + y = 0$, $x_0 = 2$

2. Determine the singular points of the following differential equations and state whether they are regular or irregular singular points.

(a) $xy'' + (1-x)y' + xy = 0$
(b) $x^2(1-x)^2y'' + 2xy + 4y = 0$
(c) $(1-x^2)^2y'' + x(1-x)y' + (1+x)y = 0$

3. Compute the Laplace transform of the following functions.

(a) $f(t) = t$

(b) $f(t) = t^n$

4. Invert the following Laplace transforms.

(a) $\mathcal{L}[f] = \dfrac{3}{s^2+4}$

(b) $\mathcal{L}[f] = \dfrac{2}{s^2+3s-4}$

(c) $\mathcal{L}[f] = \dfrac{2s+2}{s^2+2s+5}$

(d) $\mathcal{L}[f] = \dfrac{2s+1}{s^2-2s+2}$

(e) $\mathcal{L}[f] = \dfrac{1-2s}{s^2+4s+5}$

5. Use the Laplace transform to solve the given initial value problem.

(a) $y'' - y' - 6y = 0 \quad ; \quad y(0) = 1 \quad , \quad y'(0) = -1$

(b) $y'' - 2y' + 2y = 0 \quad ; \quad y(0) = 0 \quad , \quad y'(0) = 1$

(c) $y'' - 2y' - 2y = 0 \quad ; \quad y(0) = 2 \quad , \quad y'(0) = 0$

(d) $y'' + 2y' + y = 4e^{-t} \quad ; \quad y(0) = 2 \quad , \quad y'(0) = -1$