Math 2233
Homework Set 6

1. Transform each of the following expressions into a single power series.

(a) $\displaystyle\sum_{n=1}^{\infty}(n+1)(x-1)^{n-1} + \sum_{n=0}^{\infty}n(x-1)^{n}$

(b) $\displaystyle\sum_{n=0}^{\infty}(n+1)a_{n+2}x^{n+1} + \sum_{n=0}^{\infty}na_{n}x^{n-1}$

(c) $\displaystyle(x-1)\sum_{n=0}^{\infty}na_{n}x^{n-1} + \sum_{n=0}^{\infty}a_{n}x^{n}$

(d) $\displaystyle x\sum_{n=0}^{\infty}na_{n}(x-1)^{n-1} + \sum_{n=0}^{\infty}a_{n}(x-1)^{n}$

(e) $\displaystyle x^{2}\sum_{n=0}^{\infty}n(n-1)a_{n}(x-1)^{n-2}$

2. Find the recursion relations for the power series solutions $y(x) = \sum_{n=0}^{\infty} a_n(x - x_o)^n$ of the following differential equations

(a) $y'' - xy' - y = 0$, $x_o = 0$
(b) $y'' - xy' - y = 0$, $x_o = 1$
(c) $(1-x)y'' + y = 0$, $x_o = 0$
(d) $y'' + xy' + 2y = 0$, $x_o = 0$
(e) $(1+x^2)y'' - 4xy' + 6y = 0$, $x_o = 0$

3. Find power series expressions for the general solutions of the following differential equations. (You may utilize recursion relations found in Problem 2.)

(a) $y'' - xy' - y = 0$, $x_o = 0$
(b) $y'' - xy' - y = 0$, $x_o = 1$

4. Find power series expressions for the solutions to the following initial value problems. (You may utilize recursion relations found in Problem 2.)

(a) $(1-x)y'' + y = 0$, $y(0) = 2$, $y'(0) = 1$
(b) $y'' - xy' - y = 0$, $y(1) = 1$, $y'(1) = 2$