Math 2233 Homework Set 5

1. Solve the following Euler-type equations.

(a)
$$x^2y'' + xy' + y = 0$$

(b)
$$x^2y'' - xy' + 2y = 0$$

(c)
$$4x^2y'' - 4xy' + 3y = 0$$
, $y(1) = 0$, $y'(1) = 1$

(d)
$$x^2y'' - 3xy' + 3y = 0$$

(e)
$$x^2y'' + 5xy' + 5y = 0$$

2. Given that $y_1(x) = e^x$ and $y_2(x) = x$ are solutions of

$$(1 - x)y'' + xy' - y = 0$$

find the general solution to

$$(1-x)y'' + xy' - y = 2(x-1)^2 e^{-x}$$

by the method of Variation of Parameters.

3. Find the general solution of each of the following equations by the method of Variation of Parameters. If initial conditions are given, find the solution satisfying that initial value problem.

(a)
$$y'' - 3y' + 2y = 10$$

(b)
$$y'' + y = \sin(x)$$
, $y(0) = 1$, $y'(0) = 2$

(c)
$$y'' - 7y' + 10y = 100x$$

(d)
$$x^2y'' - 5xy' + 9y = x^3$$