Math 2233
Homework Set 1

1. Determine the order of the following differential equations, whether or not the equations are linear and whether the differential equations are ODEs (ordinary differential equations) or PDEs (partial differential equations).

(a) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 2y = \sin(x)$

(b) $(1+y^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = e^x$

(c) $\dfrac{\partial^2\phi}{\partial x^2} + y^2\dfrac{\partial\phi}{\partial x} = x^2$

(d) $\dfrac{d^4y}{dx^4} + \dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 1$

(e) $\dfrac{dy}{dx} + xy^2 = 0$

(f) $\dfrac{\partial^2\phi}{\partial x\partial y} + \dfrac{\partial\phi}{\partial x}\phi = x^2$

(g) $\dfrac{d^2y}{dx^2} + \sin(x+y) = \sin(x)$

2.

(a) Plot the direction field for the differential equation

$$y' = y^{4/5}.$$

(b) Sketch the solution that satisfies $y(0) = 2$.

(c) Sketch the solution that satisfies $y(0) = 1$.

3. Plot the direction fields for the following differential equations on the given interval.

(a) $y' = 2y$; $-2 \le x \le 2$, $0 \le y \le 6$.

(b) $y' = 3y(1-y)$; $-2 \le x \le 3$, $-3 \le y \le 4$.

4. Consider the differential equation $y' = (y+2)(y-2)$. What can you say about the behavior of solution $y(x)$ that passes through the point $x = 0$, $y = -1$ as $x \to \infty$? (Hint: the sign of the right hand side of the differential equations tells you whether or not a solution y(x) is increasing or decreasing.)

5. Find the general solution of the following differential equations

(a) $y' = x^2 - x + 1$

(b) $y' = x^3\sqrt{x^2+1}$

(c) $y' = x\csc^2\left(x^2\right)$

(d) $y' = \frac{1}{x^2+x}$

(e) $y' = 2xe^{3x}$

Math 2233
Homework Set 1 Supplement

Below are two sample problems that indicate how to sketch a solution using Maple and how to find a numerical solution using Maple.

1. Sketch the direction fields of the

$$\frac{dx}{dt} = t\sin(x).$$

in the region $0 \leq t \leq 2$, $0 \leq x \leq 2$.

A plot of the direction fields for this differential equation can be produced by Maple using the following commands:

```
with(DEtools);
dfieldplot(diff(x(t),t) = t*sin(x(t)),x(t),t=0..2,x=0..2);
```