Part I. Individual Round

1. Let X be the smallest positive integer whose digits (written in base 10) add to 2014. (For example, the digits of 2014 add to $2 + 0 + 1 + 4 = 7$.) How many digits does X have?

   A. 223

   B. 224

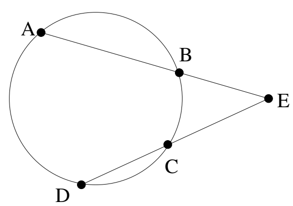
   C. 225

   D. 226

   E. 227

   Answer: B

2. In the figure below (not drawn to scale), $ABCD$ is a circle, and arcs AB, BC, and CD each measure $70°$. Find the measure of angle BEC.



   A. $10°$

   B. $15°$

   C. $20°$

   D. $40°$

   E. $80°$

   Answer: D.

3. Find the sum of all even positive integers less than or equal to 2014.

   A. 1,015,056

   B. 2,030,112

   C. 4,060,224

   D. 8,120,448

   E. 16,240,896

   Answer: A

4. Find the vertex of the parabola $y = 2x^2 - 3x + 4$.

A. $\left(\frac{3}{2}, \frac{7}{4}\right)$

B. $\left(-\frac{3}{4}, \frac{17}{2}\right)$

C. $\left(-\frac{3}{2}, 13\right)$

D. $(3, 13)$

E. $\left(\frac{3}{4}, \frac{23}{8}\right)$

Answer: E

5. Suppose a, b, and c are integers satisfying $2^a + 2^b + 2^c = 128$. Find $a + b + c$.

A. 7

B. 9

C. 12

D. 16

E. 21

Answer: D

6. How many ordered pairs of integers (a, b) satisfy $|a| + |b| \leq 2014$?

A. 8,116,421

B. 8,116,422

C. 8,116,423

D. 8,116,424

E. 8,116,425

Answer: A

7. What is the x^5 term of $(2x + y)^8$?

A. $32x^5y^3$

B. $1792x^5y^3$

C. $2014x^5y^3$

D. $336x^5y^3$

E. $320x^5y^3$

Answer: B

8. 30% of OSU students who major in math also major in education. If 1% of OSU students major in math and 10% major in education, what percentage of OSU students major in both math and education?

   A. 3%

   B. 0.03%

   C. 0.3%

   D. 0.1%

   E. 1%

   Answer: C

9. For what numbers b does the equation $x^2 - bx + 5 = 0$ have two distinct positive real roots?

   A. $b > 2\sqrt{5}$

   B. $b < -2\sqrt{5}$ and $b > 2\sqrt{5}$

   C. $b \neq \pm 2\sqrt{5}$

   D. $-2\sqrt{5} \leq b \leq 2\sqrt{5}$

   E. $-2\sqrt{5} < b < 2\sqrt{5}$

   Answer: A

10. Simplify $\dfrac{2^{2014} - 2^{2013}}{64}$.

    A. 2^{2007}

    B. 64^{2013}

    C. 2^{1001}

    D. $\dfrac{1}{32}$

    E. $2^{1950} - 2^{1949}$

    Answer: A

11. Find the remainder when $(x+5)^3$ is divided by $x+3$.

    A. -2

    B. 0

    C. 2

    D. 8

    E. 512

    Answer: D

12. How many angles θ between 0 and 2π satisfy

$$\sin\theta + \cos\theta = \frac{1+\sqrt{3}}{2}?$$

A. 0

B. 1

C. 2

D. 3

E. 4

Answer: C

13. Let $7\underline{A}\,\underline{B}5$ be a four digit number (i.e., A and B are each digits between 0 and 9). If $7\underline{A}\,\underline{B}5$ is a multiple of 99, what is $A+B$?

A. 3

B. 5

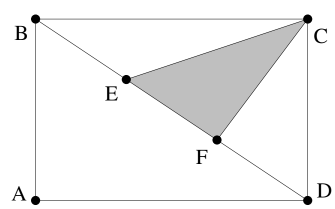
C. 6

D. 9

E. 15

Answer: C

14. In the figure below, $ABCD$ is a rectangle with sides $AB = 4$ and $BC = 6$. If E and F trisect BD (i.e., $BE = EF = FD$), find the area of triangle CEF.



A. $\dfrac{8-\sqrt{13}}{3}$

B. $\dfrac{8+\sqrt{13}}{3}$

C. 8

D. $2\sqrt{13}$

E. 4

Answer: E

15. Suppose a, b, and c are positive numbers such that $b = a^{30}$ and $c = a^{42}$. Find $\log_b(c)$.

A. $\frac{5}{7}$

B. 12

C. a^{12}

D. -12

E. $\frac{7}{5}$

Answer: E

16. A rubber ball is dropped from a height of 20 feet. Every time it hits the ground, it bounces back up 70% of its previous height. How far does the ball travel before it comes to rest? (Assume the ball only moves up and down. Thus at the top of the first bounce, it has travelled $20 + 14 = 34$ feet.)

A. $\frac{400}{3}$ feet

B. $\frac{340}{3}$ feet

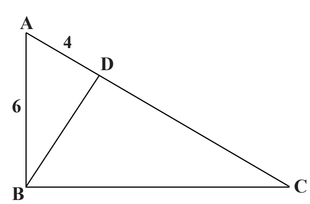
C. $\frac{260}{7}$ feet

D. 48 feet

E. $\frac{200}{7}$ feet

Answer: B

17. In the figure below, ABC is a right triangle with right angle at B, and BD is the altitude from B to AC. If $AB = 6$ and $AD = 4$, find BC.



A. $\frac{10}{3}$

B. 8

C. $2\sqrt{13}$

D. 9

E. $3\sqrt{5}$

Answer: E

18. Express the repeating decimal $.3\overline{45} = .345454545\ldots$ as a fraction in lowest terms.

A. $\dfrac{11}{30}$

B. $\dfrac{16}{45}$

C. $\dfrac{115}{999}$

D. $\dfrac{19}{55}$

E. $\dfrac{11}{33}$

Answer: D

19. Suppose α and β are acute angles satisfying $\sin\alpha + \cos\beta = \dfrac{\sqrt{3}}{2}$. What is the largest possible value of $\sin(\alpha + \beta)$?

A. $\dfrac{1}{2}$

B. $\dfrac{\sqrt{3}}{2}$

C. $\dfrac{1+\sqrt{3}}{2}$

D. 1

E. $\dfrac{1+\sqrt{3}}{4}$

Answer: D

20. How many positive integers less than 2014 have an odd number of factors?

A. 44

B. 1007

C. none

D. 1

E. 628

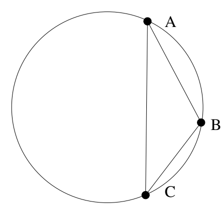
Answer: A

21. Let a_1, a_2 , a_3, and a_4 be the four (possibly complex) solutions to the quartic equation $2x^4 - 5x^2 + 2 = 0$. Find $a_1^2 + a_2^2 + a_3^2 + a_4^2$.

   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

   Answer: E

22. In the figure below, triangle ABC is inscribed in a circle of radius 1. If arcs AB and BC both measure $60°$, find the area of triangle ABC.



   A. $\dfrac{3\sqrt{3}}{4}$
   B. $\dfrac{2\pi - 3\sqrt{3}}{12}$
   C. $\dfrac{2\pi + 3\sqrt{3}}{4}$
   D. $\dfrac{4\pi - 3\sqrt{3}}{12}$
   E. $\dfrac{\sqrt{3}}{4}$

   Answer: E

23. A square has one vertex at $(1, 2)$ and another on the line $y - 3x = 4$. What is its smallest possible area?

   A. 5
   B. 4
   C. 2.5
   D. 1.25
   E. 1

   Answer: D

24. 360 cowboys attend a wedding, and each checks his hat at the door before the service begins. During the ceremony, an earthquake strikes the hat-check room, scrambling all the hats. Thus each cowboy is given a hat at random when he leaves the wedding. Now each cowboy has his name written inside his hat but with one exception they are an easygoing bunch and content to wear one another's hats.

Cowboy Bob loves his hat and has no interest in wearing another, so if he gets someone else's hat he will return it to its original owner. This fellow, having no use for two hats, will likewise return his new hat to its original owner, and the cycle will continue until eventually someone returns Cowboy Bob's hat to him.

What is the probability that Cowboy Pete ends up with his own hat? (Note that there are two ways for this to happen: Pete could get lucky and get his hat back directly after the wedding, or he could get it back as part of the cycle started by Bob.)

A. Between $\frac{2}{3}$ and 1.

B. Between $\frac{1}{2}$ and $\frac{2}{3}$.

C. Between $\frac{1}{3}$ and $\frac{1}{2}$.

D. Between $\frac{1}{120}$ and $\frac{1}{3}$.

E. Between $\frac{1}{360}$ and $\frac{1}{120}$.

Answer: B.

25. Let 1, a_1, a_2, $\ldots$, a_{2013} be the 2014 complex roots of the equation $a^{2014} = 1$. Find the product $(1 - a_1)(1 - a_2)\ldots(1 - a_{2013})$.

A. 2014

B. i

C. 1

D. 0

E. $-1007\sqrt{2} + 1006i\sqrt{2}$

Answer: A