Math 4033: History of Mathematics
Theme: A Brief History of Circles and Spheres in Mathematics
Course Syllabus
Spring 2015

Contact

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Office Hours: Monday 12pm-1:30pm and Wednesday 2pm-3pm

Prerequisites

The prerequisite for this class is Calculus II (Math 2153) or permission of the instructor.

Process Goals

- Improve your ability to clearly and correctly present and discuss mathematics orally.
- Improve your ability to write mathematics clearly and carefully.
- Be able to understand and write simple mathematical arguments. Appreciate the necessity of justifying mathematical statements.

Content Goals

- Understand the formulas for the area and circumference of a circle and the surface area and volume of a sphere (work of Euclid and Archimedes). This means carefully understanding where these formulas come from and *why* they are correct. In particular, appreciate the style of argument utilized by the ancient Greeks in their proofs of these formulas.
- Know at least one proof of the Pythagorean Theorem. Know how to find all Pythagorean triples and understand how this problem is equivalent to finding rational points on the unit circle.
- Understand Descartes' circle method for finding tangents. Be able to use it to compute tangent lines to some simple curves. Appreciate this as an early example of the usefulness of coordinate geometry in mathematics.
- Comprehend Newton's derivation of the Taylor series for the sine and cosine functions. Appreciate this both as the first formula for the x and y coordinates of a point on the unit circle in terms of the angle θ and as an example of the way Newton approached calculus (using power series and the binomial theorem with rational exponent).
- Comprehend Liebniz's method of characteristic triangles, and understand his corresponding elegant derivation of the surface area of a sphere. Recognize the thorny philosophical issues involved in Liebniz's incessant use of the fuzzy notion of an infinitesimal.
- Understand the notions of curvature of a curve and curvature of a surface. See how the circle is both the archetypical examples of a curved arc and the method of computing the curvature of an arc. Understand the principal curvatures on a surface introduced by Euler and the Gaussian curvature on a surface. Appreciate the difference between extrinsic and intrinsic quantities on surfaces in various examples including the sphere.

Requirements

- **Homework Logistics.** There will be problem sets due at the beginning of every class. Assignments will be handed out in class. *No late assignments will be accepted.* You are encouraged to work together on assignments. *However, your assignments must be written in your own words.* In addition, please write the names of everyone you worked with on the first page of your assignment. The use of outside sources (the internet, textbooks from the library, etc) for your homework assignments is strictly prohibited. *Your assignment grade will depend on the correctness of your solutions as well as the clarity of your exposition.*
- **Attendance and Class Participation.** You are required to come to class and participate in class. I will take attendance every day. I will divide the class into groups. We will often do in class group work assignments. Good participation in group work exercises will count towards your class participation grade.
- **Quizzes.** You will have a 20-30 minute in class quiz every other week. These quizzes are a way for me to make sure that you are keeping up with the material in the course. If you thoroughly understand what we are doing in class and you thoroughly understand your homework assignments, then you should have no problem passing these quizzes.
- **Final Exam.** There will be a final exam at the end of the semester which will test you on all of the material we will learn this semester in this course.
- **Oral Group Presentation.** At some point during the semester, you will be required to research a project outside of class and give an oral presentation on this topic. Groups will consist of 3 members, and group presentations will be 30 minutes long (10 minutes per person). You will choose your topic from a list that I will provide for you. Your group will have to meet with me outside of class to practice your presentation before you give it to the class.
- **Group Presentation Paper.** In addition to your oral group presentation, you will be assigned an individual sub-topic within your group's topic, and you will write a 2-3 page paper on this topic. These papers will be graded both for good mathematical content and for clear and correct exposition. You will be given an opportunity to rewrite and improve your group presentation paper.

Grading

- Attendance and Class Participation: 10%
- Homework: 25%
- In-Class Quizzes: 25%
- Oral Group Presentation: 10%
- Group Presentation Paper: 10%
- Final Exam: 20%

Grade Cutoffs

- A. 90-100
- B. 80-90
- C. 70-80
- D. 60-70
- F. 0-60

You will not be graded on a curve in this course. However, I reserve the right to "round up" grades at the end of the semester which are close to a particular cutoff.

Succeeding in this Course.

- **Office Hours.** I will hold office hours on Mondays 12pm-1:30pm and on Wednesdays 2pm-3pm. If you cannot make it to the designated office hours, we can negotiate an alternate time via email. Please schedule appointments at least 18 hours in advance.
- **Effort.** You will notice that a large portion of your grade is determined by effort. Ten percent of your grade is determined by whether you show up to class and try hard during class. Twenty five percent of your grade is determined by your performance on homework assignments. **If you are stuck on a homework assignment, come talk to me about it in office hours. "I did not know where to begin" is NOT an excuse for an incomplete homework assignment**. Ten percent of your grade is determined by an oral in class presentation. You will practice this presentation in front of me ahead of time, and I will give you feedback on it so there is no excuse for doing a poor job. Ten percent of your grade is determined by a written paper. You will be allowed to rewrite this paper after receiving my feedback on it so there is no excuse for doing a poor job.
  My grading method in this course is designed so that you can pass if you try hard. I do not believe in As for effort; I do believe in Cs for effort.

Suggested Reading

There is no textbook for this course. However, if you would like to do some further reading, here are a few recommendations.

1. *A History of Mathematics: An Introduction* by Victor J. Katz.
   A readable overview of the history of mathematics that is equal parts history and mathematics.
2. *Mathematics and Its History* by John Stillwell.
   A more advanced history of mathematics that focuses more on the mathematics than the history.
3. *The Historical Development of the Calculus* by C.H. Edwards.
   My favorite book on the history of mathematics. It only covers the history of calculus and it is not an easy read, but there is much to learn from this book.

Disclaimer

I reserve the right to alter any of these policies at any time during the semester.

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This schedule will likely be modified over the course of the semester.

1/13: Approximating π by Exhausting the Circle
1/15: The Axiom of Eudoxus *(HW 1 due)*
1/20: The Area of a Circle *(HW 2 due)*
1/22: **Quiz 1** *(HW 3 due)*
1/27: Archimedes and Approximations of π
1/29: The Circumference of a Circle *(HW 4 due)*
2/3: Archimedes and the Volume of a Sphere *(HW 5 due)*
2/5: **Quiz 2** Archimedes and the Surface Area of a Sphere *(HW 6 due)*
2/10: Proofs of the Pythagorean Theorem *(HW 7 due)*
2/12: Pythagorean Triples and Rational Points on the Unit Circle *(HW 8 due)*
2/17: Pythagorean Triples and Rational Points on the Unit Circle *(HW 9 due)*
2/19: **Quiz 3** Mathematics Between 200 BC and 1500 AD *(HW 10 due)*
2/24: **Student Presentations (Groups I and II)** *(HW 11 due)*
2/26: Descartes' Circle Method
3/3: Descartes' Circle Method *(HW 12 due)*
3/5: **Quiz 4** Pascal's Triangle *(HW 13 due)*
3/10: **Student Presentations (Groups III and IV)** *(HW 14 due)*
3/12: The Binomial Theorem with Rational Exponent
3/17: Spring Break, No Class.
3/19: Spring Break, No Class.
3/24: Newton's Method and Inversion of Power Series
3/26: **Quiz 5** Newton's Method and Inversion of Power Series *(HW 15 due)*
3/31: Newton's Series for Sine and Cosine *(HW 16 due)*
4/2: **Student Presentations (Groups V and VI)** *(HW 17 due)*
4/7: Liebniz, Infinitesimals, and Characteristic Triangles
4/9: **Quiz 6** Liebniz, Infinitesimals, and Characteristic Triangles *(HW 18 due)*
4/14: **Student Presentations (Groups VII and VIII)** *(HW 19 due)*
4/16: The Curvature of Plane Curves
4/21: **Student Presentations (Groups IX and X)** *(HW 20 due)*
4/23: **Quiz 7** The Principal Curvatures on a Surface
4/28: Gaussian Curvature *(HW 21 due)*
4/30: Review for the Final Exam *(HW 22 due)*
5/7: **Final Exam** (2:00pm-3:50pm)