Paper Folding:

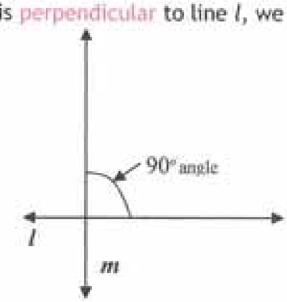
Two Basic Constructions
And Why They Work

By Janell Eck

Perpendicular Bisector of a Line Segment

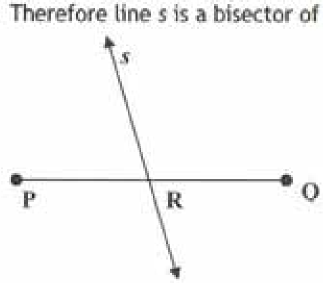
- **What does Perpendicular mean?**

  Two lines that form a right angle (90°) are called perpendicular lines.
  This is often written as ⊥.
  For example, if line *m* is perpendicular to line *l*, we would write "$m \perp l$"



- **What is a Bisector?**

  "Bisect" means to cut into 2 equal parts. So the bisector of a line is a line which divides a line segment into two equal parts.
  In the illustration below line *s* meets line segment PQ at point R. We can see that the distance between P and R is the same as the distance between R and Q. Therefore line *s* is a bisector of line segment PQ.



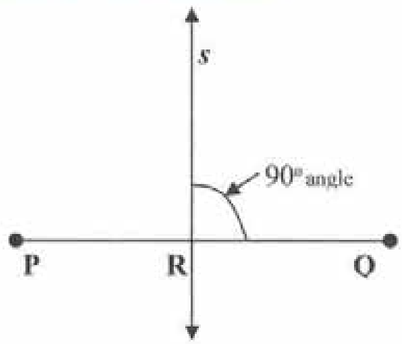
- **Line Segment? What do you mean?**

  Well, a line really never ends. It extends forever in both directions. A line segment is a part of a line but has defined beginning and end points. We can tell the difference between a line and a line segment because a line has arrows on the ends and a line segment has dots.

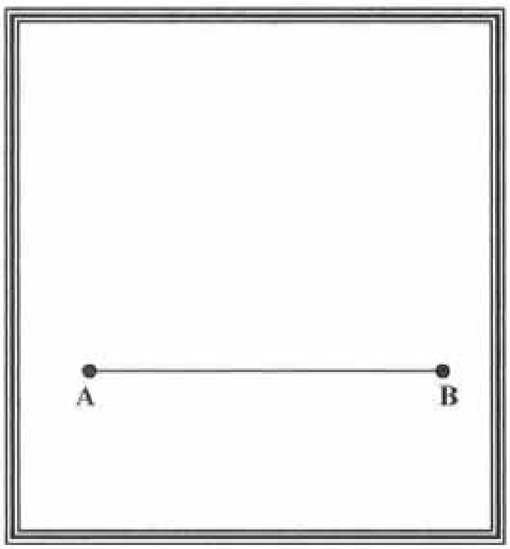
- So…we combine these to form a Perpendicular Bisector of a Line Segment.

  The perpendicular bisector of a line segment is a line that cuts a line segment exactly in half. Also, the line and the line segment form a right angle.

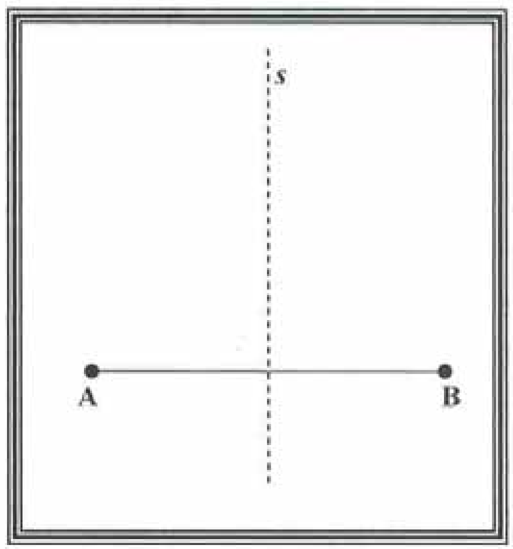
  In this example, line *s* cuts line segment PQ exactly in half at point R. Also, line *s* makes a right angle with line segment PQ. Hence, line *s* is a Perpendicular Bisector of Line Segment PQ.



Paper folding description of a Perpendicular Bisector of a Line Segment



Step 1

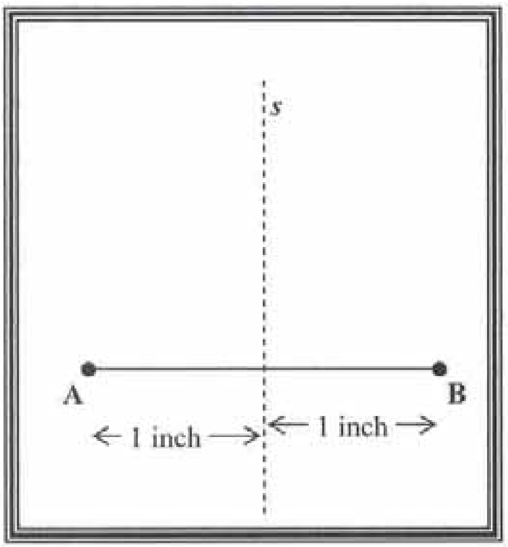


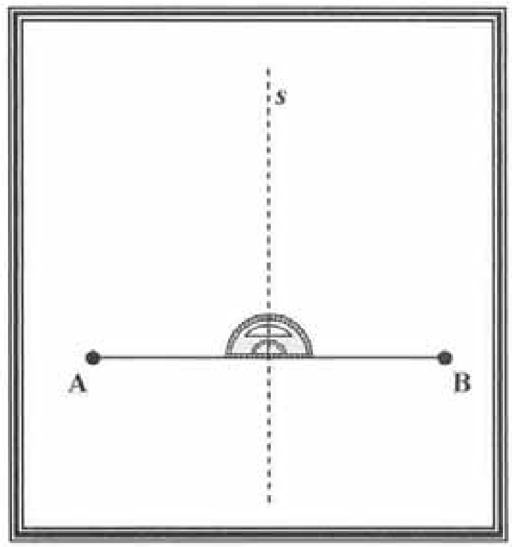
Step 2 & Step 3

1. Start with line segment AB.
2. Fold the line segment on top of itself so that point A meets up exactly with point B.
3. Now make a crease in the paper. Label this crease as line *s* (marked here as a dashed line).
4. Line *s* is the Perpendicular Bisector of Line Segment AB.

How do we know it is a Perpendicular Bisector?

The easiest way to show that line *s* really is the Perpendicular Bisector of Line Segment **AB** is by simply measuring.



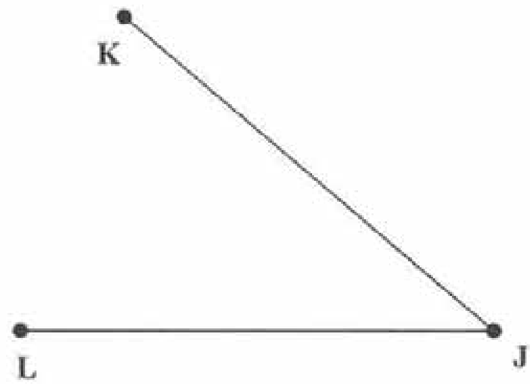


1. To see if *s* is a bisector, measure from **A** to where *s* intersects **AB** and from **B** to where *s* intersects **AB**. In our case this distance is 1 inch on each side. That means *s* is a bisector of line segment **AB**.
2. The last thing we need to check is if *s* forms a right angle with **AB**. By laying a protractor over this angle, we see that indeed *s* is perpendicular to **AB**.
3. We conclude that *s* is perpendicular bisector of line segment **AB**.

Bisector of an Angle

❖ **What is an Angle?**

An angle is a figure formed by two line segments that extend from the same point. The point where the line segments meet is called a vertex. In the illustration below, line segments **KJ** and **JL** form an angle with vertex **J**. We call this angle "Angle **KJL**" and write it as $\angle \mathbf{KJL}$.

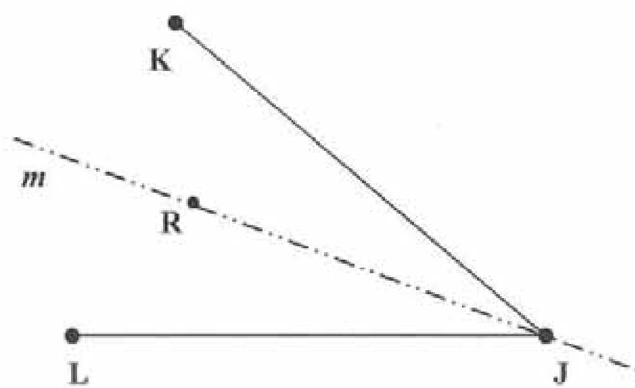


❖ **What is a Bisector?**

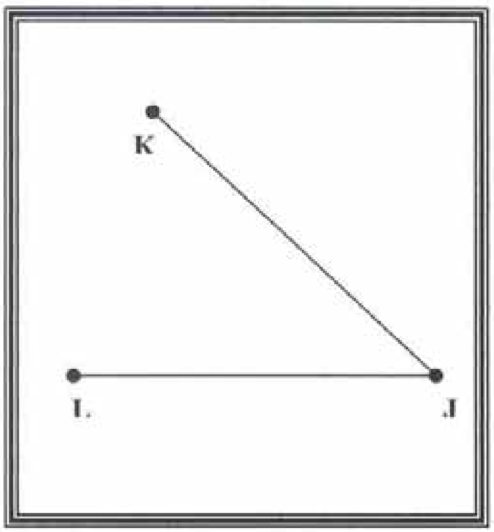
A bisector is a line that cuts something into two equal parts. In this case we are talking about an angle, so the bisector goes through the vertex.

❖ **So, we combine these terms to get the Bisector of an Angle**

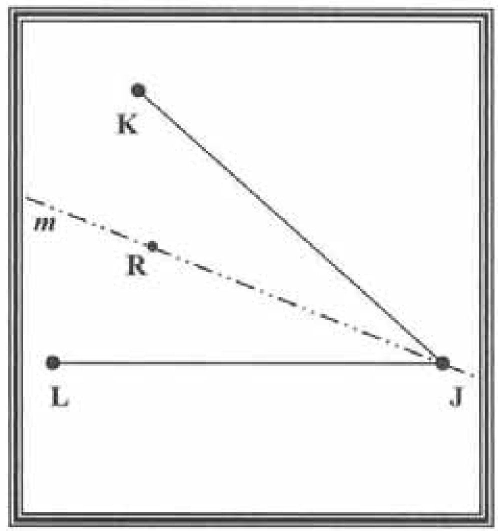
The Bisector of an Angle cuts an angle in half. We see in the picture below that line *m* is the bisector of $\angle \mathbf{KJL}$. This means that $\angle \mathbf{KJL}$ is split in half by line *m* so $\angle \mathbf{KJR}$ is equal to $\angle \mathbf{RJL}$ and $\angle \mathbf{KJR} + \angle \mathbf{RJL} = \angle \mathbf{KJL}$.



Paper folding description of a Bisector of an Angle



Step 1

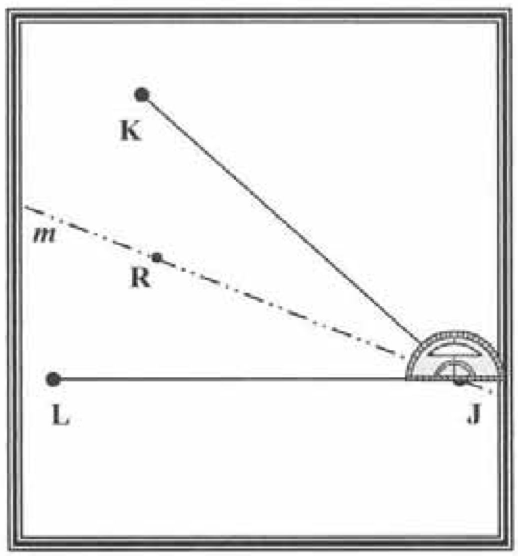


Step 2 & Step 3

1. Start with $\angle KJL$.
2. Fold line segment **KJ** on top of **JL** so that the folded line goes through the point J.
3. Now make a crease in the paper. Label this crease as line *m*.
4. Line *m* is the Bisector of Angle KJL.

How do we know it is a Bisector of an Angle?

The easiest way to show that line *m* really is the Bisector of Angle **KJL** is to measure using a protractor.



1. Measure $\angle\mathbf{KJR}$. We find that $\angle\mathbf{KJR} = 20^\circ$
2. Measure $\angle\mathbf{RJL}$. We find that $\angle\mathbf{RJL} = 20^\circ$
3. Thus $\angle\mathbf{KJR} = \angle\mathbf{RJL} = 20^\circ$.
4. Now add $\angle\mathbf{KJR}$ and $\angle\mathbf{RJL}$. So $\angle\mathbf{KJR} + \angle\mathbf{RJL} = 40^\circ$
5. Measure $\angle\mathbf{KJL}$. We see that $\angle\mathbf{KJL} = 40^\circ$
6. Therefore, we have $\angle\mathbf{KJR} + \angle\mathbf{RJL} = \angle\mathbf{KJL} = 40^\circ$ and $\angle\mathbf{KJR} = \angle\mathbf{RJL} = 20^\circ$ so we conclude that *m* really is the Bisector of Angle **KJL**.