Math 5553, Homework 5, Due on April/30/2013

1. Show that for the steepest descent method, $\mathbf{r}_i \in span\{\mathbf{r}_0, A\mathbf{r}_0, A^2\mathbf{r}_0, \ldots, A^i\mathbf{r}_0\}$ for all $i = 1, 2, \ldots$.

2. For the conjugate gradient method, prove that

$$-\frac{\mathbf{r}_{i+1} \cdot (A\mathbf{p}_i)}{(A\mathbf{p}_i) \cdot \mathbf{p}_i} = \frac{\mathbf{r}_{i+1} \cdot \mathbf{r}_{i+1}}{\mathbf{r}_i \cdot \mathbf{r}_i}$$

3. Consider the system $A\mathbf{x} = \mathbf{b}$ where the 10×10 symmetric tridiagonal matrix A and vector $\mathbf{b}$ are given by

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 2 & 0 & \cdots & 0 & 0 \\ & & & \ddots & \ddots & & \\ & & & \ddots & \ddots & & \\ 0 & 0 & 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

It is not hard to see that the exact solution $\mathbf{x} = [1\ 1\ \ldots\ 1]^t$. Set the initial guess $\mathbf{x}_0 = \mathbf{0}$.

(a) Solve the system using the Gauss-Seidel method with stopping criteria $\|\mathbf{r}_i\| < 10^{-6}\|\mathbf{r}_0\|$. How many iterations are needed? Print out the solution $\mathbf{x}_i$ at the last iteration step.

(b) Solve the system using the conjugate gradient method with stopping criteria $\|\mathbf{r}_i\| < 10^{-6}\|\mathbf{r}_0\|$. How many iterations are needed? Print out the solution $\mathbf{x}_i$ at the last iteration step.