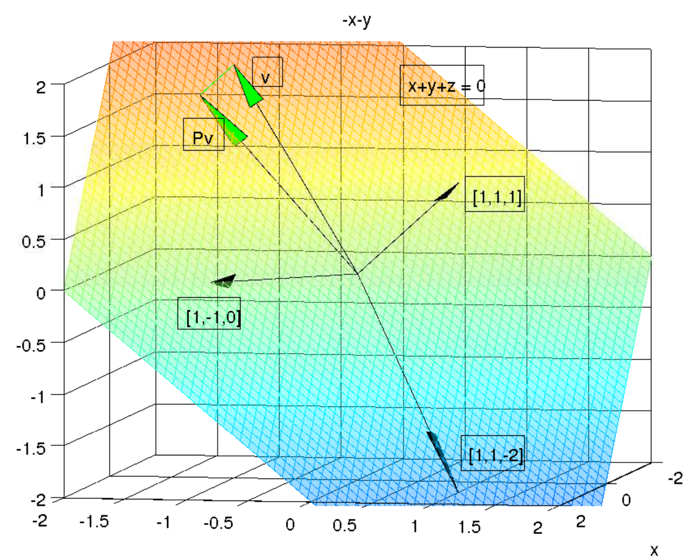
Math 5553, Homework 2, Due on 2/16/2012

1. (6 points) Define a 3×3 real-valued orthogonal projector P that maps all 3-dimensional real-valued vectors onto the plane $x + y + z = 0$, as shown in the following graph. (Hint: there are different ways to consider this problem. You can view the plane as orthogonal to the vector $[1, 1, 1]^t$. Or you can view the plan as $span\{[1, -1, 0]^t, [1, 1, -2]^t\}$. In the following graph, vectors $[1, -1, 0]^t$, $[1, 1, -2]^t$ and $P\mathbf{v}$ lie in the plane. Vector $\mathbf{v}$ is outside of the plane and vector $[1, 1, 1]^t$ is orthogonal to the plane.)



2. (6 points) Prove that vectors $\mathbf{q}_1$, $\mathbf{q}_2$, ... $\mathbf{q}_n$ obtained through the Gram-Schmidt process are orthonormal.

3. (8 points) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

where ε is a positive real number smaller than the machine epsilon. In finite precision arithmetic, we know that 1 plus/minus any positive real number smaller than the machine epsilon will still be equal to 1. We assume all other operations follow the usual arithmetic rules. For example, the Euclidean length of $[\varepsilon, \varepsilon, 0, 0]$ is $\sqrt{2}\varepsilon$. However, to simulate the finite precision arithmetic, the Euclidean length of $[1, \varepsilon, 0, 0]$ should be $\sqrt{1 + \varepsilon^2} = \sqrt{1} = 1$.

Use the classical Gram-Schmidt process and the modified Gram-Schmidt process to compute the reduced QR factorization of matrix A. Do they return the same results? Which one is the correct answer?

(The following is not part of this assignment. You do not need to do it. Remember that in exact arithmetic, the classical and the modified GS methods are the same. If you are interested, try to figure out why the classical GS method fails for this problem in finite precision arithmetic, while the modified method still works. A good strategy is to first compute the QR factorization under exact arithmetic, and then compare the results.)